

# HOUSING DYNAMICS

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## Abstract

We calibrate a dynamic model of housing in the spatial equilibrium tradition of Rosen and Roback to see whether such a model can fit the key empirical moments of the housing market such as the volatility and serial correlation of price changes and new construction. With reasonable parameter values, the model generates the empirically observed mean reversion of prices over five year periods, but cannot explain the observed positive serial correlation at higher frequencies. The model predicts the positive serial correlation of new construction that we see in the data and the volatility of both prices and quantities in the typical market. Plausible heterogeneity in construction cost parameters can readily account for the substantial variation in construction intensity across markets. However, the model cannot explain the price change volatility in the most expensive markets.

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## I. Introduction

Households hold \$18 billion dollars worth of real estate<sup>1</sup>, which represents two-thirds of the typical portfolio (Tracy, Schneider, and Chan, 1999). Despite the enormous size of this sector, there is a great deal about it that we do not understand, including the recent run-up in prices.<sup>2</sup> Four key stylized facts pose serious challenges for any model of housing markets.

First: price changes are predictable (Case and Shiller, 1989; Cutler, Poterba, and Summers, 1991). In our sample of 115 metropolitan areas from 1980 to 2005 for which we have Office of Federal Housing Enterprise Oversight (OFHEO) constant quality house price series, a \$1 increase in real house prices in one year is associated with a 71 cent increase the next year. A \$1 increase in local market prices over the past five years is associated with a 32 cent decrease over the next five year period. Can the high frequency momentum and low frequency mean reversion of price changes be reconciled with a rational market?

Second: price changes and construction levels are quite volatile in many markets. The standard deviation of three-year real changes in our sample of metropolitan area average house prices is \$26,747 (in 2000 dollars throughout the paper), which is about one-fifth of the median price level. New construction within markets also is volatile, with its standard deviation always exceeds its mean over one, three, and five year intervals. Can this volatility be the result of real shocks to housing markets or must it reflect bubbles or animal spirits?

Third: over longer time periods, price changes mean revert while quantity changes persist, as shown in Figures 1 and 2. Figure 1 depicts the significant negative correlation between house price appreciation across the 1980s and 1990s; Figure 2 documents the significant positive correlation of housing unit growth over the same two periods. This is at odds with demand-driven housing models that seem to suggest prices and quantities should move more symmetrically.

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<sup>1</sup> This figure includes primary homes, second homes, vacant homes for sale, and vacant land. See Table B.100 Balance Sheet of Households and Nonprofit Organizations at <http://www.federalreserve.gov/RELEASES/z1/Current/data.htm>.

<sup>2</sup> See McCarthy and Peach (2004), Himmelberg, Mayer and Sinai (2005), and Smith and Smith (2006) for recent analyses that conclude there is no large-scale bubble in housing prices. Shiller (2005, 2006) and Baker (2006) argue to the contrary that the bubble is both real and very large.

Fourth: most variation in housing price changes is local, not national. Less than eight percent of the variation in price levels and barely more than one-quarter of the variation in price changes across cities can be accounted for by national year-specific fixed effects.<sup>3</sup> The large amount of local variation and its relationship with macroeconomic variables such as interest rates or national income is another challenge for a consistent economic explanation of housing market dynamics.

This paper presents a dynamic, rational expectations model of house price formation to see whether such a framework can explain these facts about housing prices and quantities. The model follows the urban tradition of Alonso (1962), Rosen (1979) and Roback (1982) in which housing prices reflect the willingness to pay to live and work in a particular location. In this approach, housing prices are determined endogenously by local wages and amenities, so that local heterogeneity is natural. Our model extends the Alonso-Rosen-Roback framework by focusing on high frequency price dynamics and by incorporating endogenous housing supply.

In Section II, we present the model and its implications. The model shows that predictable housing price changes are perfectly compatible with a no-arbitrage rational expectations equilibrium. Slow construction responses to shocks and mean reverting income shocks imply that housing prices will mean revert. Enough positive serial correlation of labor demand shocks at high frequencies can generate positive serial correlation of housing prices.

The model can also explain the apparent puzzle of mean reverting prices and persistent quantity changes. Proposition 4 shows that long-term trends to city productivity or local amenities will create persistence in population and housing supply changes, but will have a much smaller impact on prices, since those trends are anticipated and incorporated into initial prices. Price changes are driven by unexpected high frequency shocks, which themselves mean revert, while quantity changes are driven by anticipated low frequency trends that persist.

The model also serves as the basis for the calibrations discussed in Sections III and IV of the paper. Section III presents our estimates of the model's key parameters: the

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<sup>3</sup> The regression results underlying these claims are provided in the appendix.

real rate of interest, the degree to which construction responds to higher prices, and the variance and serial correlation of local demand shocks.

In Section IV, we compare the moments predicted by the model based on the parameter estimates from Section III with the variances and serial correlations of actual price and quantity changes in our 115 metropolitan area sample. We do not focus on other relationships involving correlations with price changes that have been studied in some real estate and urban economics research because our analysis finds those correlations to be very sensitive to slight variations in the underlying information structure. For example, the model predicts a 98 percent correlation coefficient between income changes and housing price changes if individuals know about income shocks only when they happen, but only a 10 percent correlation coefficient if people learn about income shocks a year ahead of time. Since we have little idea about the precise timing of new information, we do not think that high frequency correlations between housing prices and other variables provide reliable information for testing any rational housing price model.

The parameter values described in Section III predict that housing prices will mean revert over five year periods at almost exactly the same rate that we see in the data. This mean reversion is due to the mean reversion of economic shocks to local productivity, with lagged responses of new construction also playing a role in those areas with more elastic housing supply. We fit the modest mean reversion of construction quantities at this lower frequency less perfectly, but the patterns in the real data still are quite compatible with reasonable parameter values.

Over one year periods, we predict a strong positive serial correlation of new construction, but the true correlation observed in the data is even greater than the level that our model predicts. Price changes also exhibit positive serial correlation at one- and three-year intervals, but our model fails to match this pattern, predicting at least modest mean reversion under any reasonable parameterization. One explanation for this fact is that the observed positive serial correlation is due to the artificial smoothing of the underlying data. Persistence itself is not enough to reject a rational expectations model, but the mismatch between the data and model at annual frequencies indicates that Case and Shiller's (1989) conclusion regarding inefficiency could be right. Future work needs

to deal with the data smoothing problem to see whether the actual serial correlation still is far too high relative to the model.

Reasonable parameter values predict variances of new construction and price changes that are quite close to the variances seen in the median metropolitan area in our sample. The model also does a reasonably good job accounting for the extensive heterogeneity in new construction intensity across markets. While our model can fit the quantity changes observed in housing markets, the same cannot be said for the variation in price changes. The second major shortcoming of the model is that it does not fit the price volatility observed in many coastal markets (California especially), which have huge price changes.

We consider four additional potential sources of volatility that might explain these coastal markets: amenity fluctuation, local taxes, unmeasured income volatility, and volatile interest rates. The one high frequency amenity variable that we have—crime rates—shows little ability to increase predicted demand and price variability. The same conclusion holds for the volatility of state taxes. Unmeasured income volatility is more interesting. Our baseline calibrations use average personal income in a market area as determined by the Bureau of Economic Analysis. Data from the Home Mortgage Disclosure Act (HMDA) files document that the volatility of incomes for recent home buyers is much higher than the volatility of average income in a market area. In addition, the variance of income in areas with big price changes also is higher than the variance of incomes for the average market. These two factors can explain much of the high variation of prices on the east coast, but not the extremely high price change volatility in coastal California markets (where measured income volatility is not especially high).

Interest rate shocks provide another possible explanation for observed behavior in those markets. Volatile interest rates will not increase the volatility of prices or construction in markets with prices close to construction costs (or to the national median price in our model), but they can increase the predicted variance in places with permanently high amenities or productivity. A preliminary analysis of this issue in Section V suggests that for interest rates to generate the high levels of volatility observed in coastal California markets, the shocks to interest rates must be extremely large and the areas must be innately extremely attractive. These markets certainly are blessed with

high quality amenities, but the required variation in rates exceeds the best estimates of the volatility of interest rate shocks. Hence, the variation of price changes in the most volatile markets remains a puzzle for rational models of housing.

## II. A Dynamic Model of Housing Prices

Our dynamic model of housing prices is based on three equilibrium conditions: (1) a spatial equilibrium that requires equal utility across markets; (2) a housing supply equilibrium condition that requires expected price to equal construction costs; and (3) a labor market equilibrium where wages equal the marginal product of labor to firms in the city.

Following Rosen (1979) and Roback (1982), we require consumers to be indifferent across space at all points in time. This implies that utility  $U(W, A, R)$  is equal across space, where  $W$  refers to wages,  $A$  to amenities, and  $R$  to the flow cost of housing. A further simplifying assumption that this spatial equilibrium must hold in all periods is the housing equivalent of assuming no financial transaction costs.<sup>4</sup>

We implement the spatial equilibrium condition by assuming that there is a “reservation locale” that delivers utility of  $\underline{U}(t)$  in each period “ $t$ ” and that the cost of building a home there always equals “ $C$ ,” which reflects the physical costs of construction. Since housing always can be built in the reservation locale at cost  $C$ , we assume that the price of a house there always equals  $C$ .<sup>5</sup> The reservation locale represents the many metropolitan areas in the American hinterland with steady growth and where prices stay close to the physical costs of construction (Glaeser, Gyourko and Saks, 2005).<sup>6</sup> The annual cost of living in the reservation locale equals the difference between the price of the house at time  $t$  and the discounted value of the house at time  $t+1$ ,

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<sup>4</sup> This is a standard assumption, but future work should investigate whether it is feasible to follow the route suggested by Hansen and Jagannathan (1991) and introduce realistic transaction costs.

<sup>5</sup> While it is possible that prices will deviate around this value because of temporary over- or under-building, we simplify and assume that the price of a house always equals  $C$ .

<sup>6</sup> Van Neuwercburgh and Weill (2006) present a similar model in their exploration of long run changes in the distribution of income. Our paper was produced independently of theirs, and our focus on high frequency variation in prices and quantities is quite different from their focus on changes in the long run distribution of housing prices. More generally, the approach taken here differs from most research into housing prices, which employs the user cost approach introduced by Hendershott and Slemrod (1983) and Poterba (1984). That branch of the literature is too voluminous to describe in detail. The first three papers referenced in footnote 2 employ a user cost framework to examine the recent housing boom.

or  $C - C/(1+r) = rC/(1+r)$ , where  $r$  is the assumed fixed rate of interest.<sup>7</sup> We abstract from taxes, maintenance costs and allow time-varying interest rates only in Section 5.<sup>8</sup>

The spatial equilibrium requires all cities at all times to deliver to the marginal resident the same expected utility as in the reservation locale. For simplicity, we focus on the dynamics in a single representative city (which is different from the reservation city). The utility flow for person  $i$  living in that city during period  $t$  is  $W(i,t) + A(i,t)$ , or wages plus amenities. We assume that there are a fixed number of firms, each of which has output that is quadratic in labor. This ensures that the marginal product at each firm is linearly decreasing with the number of workers, and that wages in the city are linearly decreasing with the number of workers. These labor demand schedules generated by firm optimization underpin our assumption that wages at the city level include a stochastic time-varying component and a component that is linearly decreasing in total city population.

We assume that the time-specific and individual-specific effects that make up the net utility flow from the city are separable, so  $W(i,t) + A(i,t) - \underline{U}(t)$  can be written as  $D(t) + \theta(i)$ . The composite variable  $D(t)$  reflects wages and amenities (which in turn reflect exogenous shocks and city size), and  $\theta(i)$  is a uniformly distributed taste for living in this particular locale. We let  $N(t)$  denote the housing stock in the city and assume that the city's population and labor force equal a constant times the amount of housing.<sup>9</sup> We further assume that  $D(t)$  moves linearly with city population to allow for the fact that wages and amenities may fall due to congestion or rise because of agglomeration economies as city size increases. The value of  $\theta(i)$  for the marginal resident at time  $t$  (denoted  $\theta(i^*(t))$ ) is also linearly decreasing in locale size.

The exogenous components of city amenities and wages include a city-specific component (denoted  $\bar{D}$ ), a city-specific time trend (denoted  $qt$ ) and a mean zero stochastic component (denoted  $x(t)$ ). Thus, the flow of utility for the city's marginal

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<sup>7</sup> This difference would also be the rent that a landlord earning zero profits would charge a tenant.

<sup>8</sup> If maintenance costs are independent of housing values and constant over space, they will not change the analysis. If maintenance costs scale with housing and if there are property taxes, then the cost of owning a house would be higher than the after-tax interest rate. For this reason, we will assume a relatively high real rate in our simulations. See below for more on that.

<sup>9</sup> Glaeser, Gyourko and Saks (2006) provide evidence showing that population is essentially proportional to the size of the housing stock.

resident at time  $t$  with index  $i^*(t)$  relative to the reservation locale,  $D(t) + \theta(i^*(t))$ , can be written  $\bar{D} + qt + x(t) - \alpha N(t)$ , where  $\alpha$  captures the assumption that wages, amenities and the taste of the marginal resident for living in the locale can fall linearly with city size. We further assume that  $x(t)$  follows an auto regressive moving average (ARMA) (1, 1) process so that  $x(t) = \delta x(t-1) + \varepsilon(t) + \theta \varepsilon(t-1)$ , where  $0 < \delta < 1$ , and the  $\varepsilon(t)$  shocks are independently and identically distributed with mean zero.

The expected cost of housing in the representative locale equals  $H(t)$  minus  $E_t(H(t+1))/(1+r)$ , where  $E_t(\cdot)$  denotes the time  $t$  expectations operator. The difference between the cost of housing in the representative city and housing costs in the reservation locale,  $rC/(1+r)$ , should be understood as the cost of receiving the extra utility flow associated with locating in the city. If extra housing costs in the city equals extra utility delivered by the city then:

$$(1) \quad H(t) - \frac{E_t(H(t+1))}{1+r} - \frac{rC}{1+r} = \bar{D} + qt + x(t) - \alpha N(t).$$

Equation (1) represents a dynamic version of the Rosen-Roback spatial indifference equation, in which differences in housing costs equal differences in wages plus differences in amenities. We assume a transversality condition on housing prices so that

$$\lim_{j \rightarrow \infty} \left( \frac{H(t+j)}{(1+r)^j} \right) = 0.^{10}$$

While our housing market equilibrium assumption that price equals construction costs requires the city to be growing, important insights can be gleaned by briefly considering the case in which housing supply is fixed, so  $N(t)=N$  (as might be the situation for a declining city as analyzed in Glaeser and Gyourko, 2005). In that case,

$$(2) \quad H(t) = C + \frac{(1+r)(\bar{D} - \alpha N + qt)}{r} + \frac{(1+r)q}{r^2} + \frac{(1+r)x(t) + \theta \varepsilon(t)}{1+r-\delta}.$$

Housing prices are a function of exogenous population and exogenous shocks to wages and amenities. The derivative of housing prices with respect to a one dollar permanent increase in wages will be  $(1+r)/r$ . Note that house price changes are predictable in this framework as long as there are predictable components to changes in urban wages and

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<sup>10</sup> This assumption limits the possible role of housing bubbles. While our focus here is on a purely rational model, we expect that future work will consider dropping this assumption.

amenities. The ARMA(1,1) structure of the shocks makes it possible to have the positive correlation of changes at high frequencies and the negative correlation at lower frequencies that we see in the data.

In our model, however, the city grows so that  $N(t)$  equals  $N(t-1) + I(t)$ , where  $I(t)$  is the amount of construction in time  $t$ .<sup>11</sup> The physical, administrative and land costs of producing a house are  $C + c_0t + c_1I(t) + c_2N(t-1)$ , where  $c_1 > c_2$  because current housing production should have a bigger impact on current construction costs than housing production many years ago.<sup>12</sup> Investment decisions for time  $t$  are made based on time  $t-1$  information, and there is free entry of risk neutral builders. Thus, if there is any building, construction costs will equal the time  $t$  expected housing price as described in equation (3):

$$(3) E_{t-1}(H(t)) = C + c_0t + c_1I(t) + c_2N(t-1).^{13}$$

Equations (1) and (3) together describe housing supply and demand, and together generate the steady state values of housing prices, which equals a city-specific constant plus  $\frac{(1+r)(\alpha c_0 + qc_2)}{rc_2 + \alpha(1+r)}$  times time, investment and housing stock, which are described in the Appendix and denoted  $\hat{H}(t)$ ,  $\hat{I}$  and  $\hat{N}(t)$  respectively.

If  $x(t)=0$  for all  $t$ , and  $\hat{N}(t) = N(t)$  for some initial period, then the steady state quantities would fully describe this representative city.<sup>14</sup> Secular trends in housing prices can come from trend in housing demand as long as  $c_2 > 0$ , or the trend in construction costs as long as  $\alpha > 0$ . If  $c_2 = 0$  so that construction costs don't increase with total city size, then trends in wages or amenities will impact city size but not housing prices. If  $\alpha = 0$  and city size doesn't decrease wages or amenities, then trends in construction costs will impact city size but not prices.

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<sup>11</sup> For simplicity, we do not allow depreciation, which is not unreasonable for shorter term housing dynamics, but would not be appropriate for a very long term analysis of city population changes.

<sup>12</sup> We deviate from the investment cost assumptions of Topel and Rosen (1988) by assuming that costs are increasing with the total level of development and not with changes in the level of investment.

<sup>13</sup> The model can be extended to allow for the possibility that, in some states of the world, new construction will be zero. This adds much complication and only a modest amount of insight into our questions.

<sup>14</sup> In this case, the assumption that there is always some construction requires that  $q(1+r) > rc_0$ .

Proposition 1 then describes housing prices and investment when there are shocks to demand and when  $\hat{N}(t) \neq N(t)$ . All proofs are in the appendix.

*Proposition 1:* At time  $t$ , housing prices equal

$$H(t) = \hat{H}(t) + \frac{\bar{\phi}}{\bar{\phi} - \delta} x(t) + \frac{\theta}{\bar{\phi} - \delta} \varepsilon(t) - \frac{\alpha(1+r)}{1+r-\phi} (N(t) - \hat{N}(t))$$

and investment equals

$$I(t+1) = \hat{I} + \frac{(1+r)}{c_1(\bar{\phi} - \delta)} (\delta x(t) + \theta \varepsilon(t)) - (1-\phi)(N(t) - \hat{N}(t)),$$

where  $\bar{\phi}$  and  $\phi$  are the two roots of

$$c_1 y^2 - ((2+r)c_1 + (1+r)\alpha - c_2)y + (1+r)(c_1 - c_2) = 0 \text{ and } \bar{\phi} \geq 1+r \geq 1 > \phi \geq 0.$$

This proposition describes the movement of housing prices and construction around their steady state levels. A temporary shock,  $\varepsilon$ , will increase housing prices by  $\frac{\bar{\phi} + \theta}{\bar{\phi} - \delta}$  and increase construction by  $\frac{(1+r)(\delta + \theta)}{c_1(\bar{\phi} - \delta)}$ . Higher values of  $\delta$  (i.e., more permanent shocks) will make both of these effects stronger. Higher values of  $c_1$  mute the construction response to shocks and increase the price response to a temporary shock (by reducing the quantity response). These results provide the intuition that places which are quantity constrained should have less construction volatility and more price volatility.

The next proposition provides implications about expected housing price changes.

*Proposition 2:* At time  $t$ , the expected home price change between time  $t$  and  $t + j$  is

$$\hat{H}(t+j) - \hat{H}(t) + \left( \frac{\alpha(1+r)}{1+r-\phi} - \phi^{j-1}((1-\phi)c_1 - c_2) \right) (N(t) - \hat{N}(t)) - x(t) + \frac{1}{\bar{\phi} - \delta} \left( \frac{1+r}{c_1} \frac{\delta^{j-1}((1-\delta)c_1 - c_2) - \phi^{j-1}((1-\phi)c_1 - c_2)}{\phi - \delta} - 1 \right) E_t(x(t+1)),$$

the expected change in the city housing stock between time  $t$  and  $t + j$  is

$$j\hat{I} + \frac{1+r}{c_1(\bar{\phi} - \delta)} \frac{\phi^j - \delta^j}{\phi - \delta} E_t(x(t+1)) - (1-\phi^j)(N(t) - \hat{N}(t)),$$

and expected time  $t + j$  construction is

$$\hat{I} + \frac{1+r}{c_1(\bar{\phi} - \delta)} \left( \frac{\delta^{j-1}(1-\delta) - \phi^{j-1}(1-\phi)}{\phi - \delta} \right) E_t(x(t+1)) - \phi^{j-1}(1-\phi)(N(t) - \hat{N}(t)).$$

Proposition 2 delivers the implication that a rational expectations model of housing prices is fully compatible with predictability in housing prices. If utility flows in a city are high today and expected to be low in the future, then housing prices will also be expected to decline over time. Any predictability of wages and construction means that predictability in housing price changes will result in our model.

The predictability of construction and prices comes in part from the convergence to steady state values. If  $x(t) = \varepsilon(t) = 0$  and initial population is above its steady state level, then prices and investment are expected to converge on their steady state levels from above. If initial population is below its steady state level and  $x(t) = \varepsilon(t) = 0$ , then price and population are expected to converge on their steady state levels from below.

The rate of convergence is determined by  $r$  and the ratios  $\frac{c_1}{\alpha}$  and  $\frac{c_2}{\alpha}$ . Higher levels of these ratios will cause the rate of convergence to slow by reducing the extent that new construction will respond to changes in demand.

The impact of a shock,  $x(t)$ , is explored in the next proposition.

*Proposition 3:* If  $N(t) = \hat{N}(t)$ ,  $x(t-1) = \varepsilon(t-1) = 0$ ,  $\theta > 0$ ,  $c_2 = 0$ , and  $\varepsilon(t) > 0$ , then investment and housing prices will initially be higher than steady state levels, but there exists a value  $j^*$  such that for all  $j > j^*$ , time  $t$  expected values of time  $t+j$  construction and housing prices will lie below steady state levels. The situation is symmetric when  $\varepsilon(t) < 0$ .

Proposition 3 highlights that this model not only delivers mean reversion, but overshooting. Figure 3 shows the response of population, construction and prices relative to their steady state levels in response to a one time shock. Construction and prices immediately shoot up, but both start to decline from that point. At first, population rises slowly over time, but as the shock wears off, the heightened construction means that the city is too large relative to its steady state level. Eventually, both construction and prices end up below their steady state levels because there is too much housing in the city relative to its wages and amenities. Places with positive shocks will experience mean reversion, with a quick boom in prices and construction, followed by a bust.

Finally, we turn to the puzzling empirical fact that, across the 1980s and 1990s, there was strong mean reversion of prices and strong positive serial correlation in

population levels. We address this by looking at the one period covariance of price and population changes. We focus on one-period for simplicity, but we think of this proposition as relating to longer time periods. Since mean reversion dominates over long time periods, we assume  $\theta = 0$  to avoid the effects of serial correlation:

*Proposition 4:* If  $N(0) = \hat{N}(0)$ ,  $\theta = 0$ ,  $x(0) = \varepsilon(0)$ , cities differ only in their demand trends  $q$  and their shock terms  $\varepsilon(0)$ ,  $\varepsilon(1)$  and  $\varepsilon(2)$ , and the demand trends are uncorrelated with the demand shocks, then:

(a) the coefficient estimated when regressing second period population growth on first period population growth will be positive if and only if

$$\frac{Var(q)}{Var(\varepsilon)} > (1 - \delta - \phi) \left( \frac{\delta(rc_2 + \alpha(1+r))}{c_1(\bar{\phi} - \delta)} \right)^2, \text{ and}$$

(b) the coefficient estimated when regressing second period price growth on first period price growth will be negative if and only if  $\Omega \left( \frac{rc_2 + \alpha(1+r)}{(1+r)c_1c_2(\bar{\phi} - \delta)} \right)^2 > \frac{Var(q)}{Var(\varepsilon)}$ , where

$$\Omega = \left( \frac{\alpha(1+r)^2\delta}{1+r-\phi} + c_1(1-\delta)\bar{\phi} \right) \left( (1-\delta-\phi) \frac{\alpha\delta(1+r)^2}{1+r-\phi} + c_1(1-\delta+\delta^2)\bar{\phi} \right).$$

Proposition 4 tells us that positive correlation of quantities and negative correlation of prices are quite compatible in the model. The positive correlation of quantities is driven by the heterogeneous trends in demand across urban areas. As long as the variance of these trends is high enough relative to the variance of temporary shocks, there will be positive serial correlation in quantities as in Figure 2.

The mean reversion of prices is driven by the shocks, and as long as  $c_2$  is sufficiently low, prices will mean revert. As discussed above, when  $c_2$  is low, trends will have little impact on steady state price growth. The positive trends show up mainly in the level of prices not in the growth of prices. However, regardless of the value of  $c_2$ , unexpected shocks impact prices and, if these shocks mean revert, then so will prices. This suggests two requirements for the observed positive correlation of quantities and negative correlation of prices: city-specific trends must differ significantly and the impact of city size on construction costs must be small.

The extensive heterogeneity in city-specific trends is discussed and documented by Gyourko, Mayer, and Sinai (2006) and Van Nieuwerburgh and Weill (2006). The

literature on housing investment suggests that the impact of city size on construction costs is small (Topel and Rosen, 1988; Gyourko and Saiz, 2006). Thus, we shouldn't be surprised to see positive serial correlation in quantity changes and negative serial correlation in price changes.

### III. Key Parameter Values for the Calibration Exercises

We now use the model as a calibration tool to see what moments of the data can and cannot be explained by our framework. We focus on the movements in prices and construction intensity around steady state levels. The model's predictions depend on seven parameters: the real interest rate ( $r$ ), the degree to which demand declines with city population ( $\alpha$ ), the degree to which construction responds to higher costs ( $c_1$  and  $c_2$ ), the time series pattern of local economic shocks ( $\delta$  and  $\theta$ ), and the variation of those shocks ( $\sigma_\varepsilon^2$ ). Table 1 reports the value of these parameters which are used in the calibration exercise, with the remainder of this section discussing how we estimate or impute them.

#### *The Real Interest Rate ( $r$ )*

The first row of Table 1 shows that we use a real interest rate ( $r$ ) of 4 percent in all calibrations. This value is higher than standard estimates of the real rate because it is also meant to reflect other facets of the cost of owning, such as taxes or maintenance expenses, that might scale with housing. The core simulation results are robust to a wide range of alternative values of  $r$  (e.g., from 2.5-5 percent).

#### *Supply Side Parameters: $c_1$ and $c_2$*

The parameter  $c_1$  reflects the extent that construction costs—which include land assembly, permitting and physical construction costs—rise with the level of current construction activity. The  $c_2$  parameter measures the sensitivity of costs to the level of overall development, or market size. The literature provides us with little guidance on the critical housing cost parameters. Moreover, these parameters surely differ significantly between metropolitan areas like Las Vegas, which is pro-growth, and Greater Boston, which isn't (Glaeser and Ward, 2006), because of differences in both

physical construction costs and local land use regulation (Gyourko and Saiz, 2006; Gyourko, Saiz and Summers, forthcoming). Both uncertainty and heterogeneity across areas leads us to calibrate using a wide range of construction cost parameters.

To determine reasonable values for  $c_l$ , we begin by examining the relationship between physical construction costs and permitting levels over time for a large number of metropolitan areas. The construction cost data are taken from Gyourko and Saiz (2006) and are based on figures from the R.S. Means Company, a consultant to the homebuilding industry. The R.S. Means Company provides estimates of the costs to construct homes of given qualities. We use annual cost data from 1980-2004 for a 2,000 square foot ‘economy’ quality home that meets all building code and regulatory requirements in each market.<sup>15</sup>

The baseline specification regresses physical construction costs per square foot of a standard home on annual housing permits and a time trend which controls for the secular decline in real costs in most markets (Gyourko and Saiz, 2006). The range of parameter values reported below for  $c_l$  is based on estimations that pool across markets within the nine census divisions.<sup>16</sup>

Higher permitting activity is associated with the lowest increase in physical construction costs in the markets in the South Atlantic Mountain and West South Central divisions. For example, one thousand additional permits is associated with a \$120 increase in the cost of building a standard house in the Mountain division market and a \$140 increase in the South Atlantic and West South Central division areas.<sup>17</sup> More

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<sup>15</sup> More specifically, the R.S. Means Company assumes this standard home is built according to a common, national specification. It then disaggregates construction of this unit into different tasks that require materials and labor, and surveys local suppliers and builders to determine local prices for the inputs into each construction task. Local physical construction costs are the sum of the materials and labor costs needed to complete each task.

<sup>16</sup> We were able to obtain data on both construction costs and permits over the 1980-2004 period for 161 markets and use information from all those areas in determining the range of parameter values for  $c_l$ . If we restrict the analysis to the 115 markets for which OFHEO reports a constant quality price index since 1980, the results are not materially different. We also estimated the relationship for each metropolitan area and comment below on some of those results. As expected, the range of parameter values obtained when pooling to the census division level is smaller than that resulting from the individual metropolitan area estimations, but those differences are not great, as the variation in supply side conditions across census divisions is substantial. All underlying regression results are available upon request.

<sup>17</sup> Naturally, there is variation about that mean estimate when one looks at individual markets. While the results are not precisely estimated for each metropolitan area, the bulk of the results imply that an additional permit is associated with a 10-20 cent increase in physical constructions costs. For example, the estimate for Dallas is 7 cents, that for Phoenix is 12 cents, Atlanta is 18 cents, and Tampa is 26 cents.

permitting is associated with the largest increase in construction costs in the New England division. There, one thousand extra permits implies a \$1,900 increase in construction costs for our 2,000 square foot home.<sup>18</sup> In the Pacific census division, 1,000 extra units increases construction costs by \$3,680 for the Santa Barbara-Santa Maria metropolitan area and \$1,740, for the San Francisco area. We interpret the data as suggesting that something around \$2,000 is a reasonable upper bound for the impact of an additional 1,000 permits on physical construction costs.

These estimates suggest that each new unit is associated with increasing physical construction costs of between 15¢ and \$2, but they do not include the costs of regulatory approval or land assembly. The importance of these costs probably is quite low in high growth, low regulation markets such as Phoenix and Las Vegas, but they could be as much as three quarters of costs in some coastal markets (Glaeser and Gyourko, 2003). Hence, we report simulation results for seven values of  $c_1$  ranging from 15¢ to \$50 dollars. We think that a value of 50¢ or less is appropriate for the high unit growth markets with very few restrictions on new building activities. For the median market, we believe that a  $c_1$  value of around \$2 best captures the reality of the supply side. In high cost areas, the value of  $c_1$  could well be \$20 or more, with \$50 representing what we believe is an upper bound.<sup>19</sup>

There is even less of a literature to guide our choice of  $c_2$ , although there is no doubt that  $c_2 < c_1$  because the flow of current construction influences price more than does the flow of past construction. We assume that  $c_2$  scales with  $c_1$ , as both reflect general supply conditions in the area. More specifically, we consider a range of ratio of  $c_2$  to  $c_1$  (henceforth denoted  $\omega$ ), that includes 0.0, 0.25, and 0.50.<sup>20</sup>

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<sup>18</sup> The highest individual market estimates are for the metropolitan areas containing the Connecticut and New Jersey suburbs of New York City. They range from \$2.43 in New Haven to \$3.48 in Trenton.

<sup>19</sup> This range can be compared to values of  $c_1$  implied by the housing supply elasticities estimated by Topel and Rosen (1988). Those authors used national data and estimated a supply elasticity ranging from 1.4 to 2.2. This supply elasticity is the relationship between the logarithm of investment and the logarithm of price, which in our model equals  $H(t)/c_1 I(t)$ . Using the mean values of investment and housing prices across our cities and an elasticity of 1.8, this generates a range of  $c_1$  from 1 to 151. The median value is 18, which seems high since it implies that a thousand additional permits (which is not a large number for the typical American metropolitan area) would imply an \$18,000 increase in house cost. Hence, we prefer the lower range associated with our estimation.

<sup>20</sup> An alternative method of estimating these parameters suggests a value of 0.25 for  $\omega$ . That approach follows Rosen and Topel (1988) in inverting the construction cost equation to obtain  $I(t) = (1/c_1)(E_{t-1}[H(t) -$

### *Increases in Population and the Marginal Valuation of an Area: $\alpha$*

The value of  $\alpha$  reflects the impact that an increase in the housing stock will have on the willingness to pay to live in a locale. If population was fixed, equation (2) tells us that the derivative of housing prices with respect to the housing stock equals  $-(1+r)\alpha/r$ , which can be seen as the slope of the housing demand curve. Typically, housing demand relationships are estimated as elasticities. Consequently, we must transform estimated demand elasticities into a levels estimate by multiplying by  $r/(1+r)$ .

While many housing demand elasticity estimates are around one (or slightly below, in absolute value), there is a wide range in the literature, so we experiment with a range from 0 to 2. We begin the transformation from an elasticity to a level by multiplying by the ratio of price to population, which produces a range of estimates for  $(1+r)\alpha/r$  of from 0 to 3. Multiplying this span by  $r/(1+r)$  yields a range from 0 to 0.15. We use a parameter estimate of 0.1 in our simulations which implies that for every 10,000 extra homes sold, the marginal purchaser likes living in the area \$1,000 less per year (see row 5 of Table 1). This estimate seems high to us, but lower estimated values of  $\alpha$  do not significantly change the simulations.

### *Time Series Properties and Variance of Shocks: $\delta$ , $\theta$ , and $\sigma_e^2$*

The model does not separately address wages and amenities, but there is little evidence on high frequency changes in amenities, so we initially assume that the high frequency movement in demand is driven by changes in labor demand, not changes in the valuation of amenities. More specifically, observed wages  $W(t)$  are presumed to equal  $w_0 - \gamma N(0) + w_1 t + x(t) - \gamma(N(t) - N(0))$ , where  $w_0 - \gamma N(0)$  is a component of  $\bar{D}$ ,  $w_1$  is a component of  $q$  and  $\gamma$  is a component of  $\alpha$ . Controlling for a city-specific fixed effect

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C)  $-(c_2/c_1)(N_{t-1})$ . In empirically implementing this equation, we used total housing permits in period  $t$  to proxy for new construction in period  $t+1$ , and actual house prices to measure expected values. Obviously, the use of actual prices in lieu of expected prices introduces some bias, but it should be small since the annual time period over which price is measured is relatively short. We also imputed the housing stock  $N(t)$  each year as described below. A simple regression of each market's resulting  $c_2$  value on its  $c_1$  value (with no intercept, as suggested by our assumed functional form) yielded a coefficient of 0.25. The estimated coefficient is 0.21 if we allow for an intercept.

and trend will eliminate the term  $w_0 - \gamma N(0) + w_1 t$ , and the residual component of wages equals  $x(t) - \gamma(N(t) - N(0))$ .<sup>21</sup>

To estimate this quantity, we need to find a reasonable measure of high frequency income changes and to adjust this measure for changes in the size of the workforce. While the Bureau of Economic Analysis (BEA) per capita personal income data is not perfect for our purposes, this source includes information on all of our metropolitan areas for which we have house price data going back to 1980.<sup>22</sup> Since we are interested in household income, we multiply per capita income by 2.63, which is the average ratio of people per household in our sample in 1990.

One problem with this data is that it includes interest income and dividends, plus a variety of transfer payments (but not capital gains), while our model implies that the appropriate income series is that attributable to living in the local labor market area. At least some dividend and interest payments will not meet this standard.<sup>23</sup> A second problem is that the measure covers everyone in the market, not just potential homebuyers. This lack of selection is problematic if buyers are younger and have more volatile incomes, or if homebuyers on the margin are in a city's fastest growing and most volatile industries (e.g., finance in New York and technology in San Francisco).

To investigate this possibility, we also turned to the Home Mortgage Disclosure Act (HMDA) files that provide reported income on mortgage applications for those who applied for home loans in all markets across the United States.<sup>24</sup> We use the incomes of

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<sup>21</sup> Unfortunately, the literature on regional shocks (e.g. Blanchard and Katz, 1992) does not yield the parameter estimates that we need to calibrate the model.

<sup>22</sup> Our data series runs from 1980-2004. We do not use the recently released personal income data for 2005 because it was computed in a different way. To speed up the release of this variable, the BEA recently began extrapolating certain components of personal income by formula when source data were not immediately available. We wish to avoid the inevitable smoothing such procedures will introduce.

<sup>23</sup> The Bureau of Labor Statistics reports disaggregated wage and salary data, but we do not use it here primarily because it is only available for a small set of metropolitan areas (about 40) before the late 1980s. In addition, this source can generate some very small sample sizes for all but the largest markets even in recent years.

<sup>24</sup> The Home Mortgage Disclosure Act was enacted to monitor the behavior of Federal Depository Insurance Corporation (FDIC) member banks. The data were purchased from the FDIC. The dataset includes several variables concerning the race, sex, location, and income of the applicants for mortgage loans, as well as information concerning the amount and purpose of the loan (e.g., purchase, home improvement, refinancing). For more detail, see "Home Mortgage Disclosure Act Raw LAR and TS Public Data" (1990-2004). Federal Financial Institutions Examination Council, Board of Governors of the Federal Reserve System . 20<sup>th</sup> & C Streets, N.W. Mail Stop 502 Washington, D.C. 20551. On the web, see <http://www.ffiec.gov/hmda>.

those mortgage applicants who were approved for loans to purchase a home, and match observations to the 115 metropolitan areas in our sample. A benefit of the HMDA data set is that it provides very large samples even in small population markets.<sup>25</sup> While the HMDA data directly measures purchasers' incomes, it is also imperfect because it includes only those people who bought, not the complete pool of potential purchasers. In addition, this data source is only available after 1990 at the metropolitan level, so that it cannot be compared with the BEA series over the full sample period.

Our model indicates that any raw income series should be adjusted for changes in the labor force, so we need an estimate of  $\gamma$ , the slope of labor demand. Customarily, labor demand is estimated as an elasticity,  $\frac{Labor\ Force}{Wage} \frac{\partial Wage}{\partial Labor\ Force}$ , and most estimates of it are statistically indistinguishable from zero (e.g., Card and Butcher, 1991). Borjas (2003) finds a higher estimate of -0.3, although this is at the national level. We use this upper-bound estimate, but an estimate of zero makes almost no difference, because high frequency changes to area's population levels are small.

For our sample of metropolitan areas, the mean of BEA income in 1990 (the middle of our sample period) was \$26,965 in year 2000 dollars. Mean employment in 1990 across these metropolitan areas was 539,215, so our ratio of wage to the labor force is about 0.05 ( $\sim 26,965/539,215$ ). Based on these numbers, an elasticity of -0.3 suggests that each worker is associated with 1.5 cents less annual income in the city. In our sample, there are on average 1.26 workers per home, so each extra home is associated with 1.9 cents per year less annual income, which serves as our estimate of  $\gamma$ .

We then adjust the income measure for the size of the local market using our estimate of 0.019 for  $\gamma$ . We base our adjustments on changes in the number of households which are potentially supplying labor. To estimate annual changes in the number of households, we impute the housing stock based on decadal census estimates of the housing stock and annual permits data. Specifically, we estimate the housing stock at

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<sup>25</sup> For example, in a small market such as Akron, OH, there are about 10,000 observations per year. For larger markets such as Chicago and Los Angeles, the number of observations typically is 10-20 times larger.

time  $t+j$  to be  $N(t) + \frac{\sum_{i=0}^{j-1} Permits_{t+i}}{\sum_{i=0}^9 Permits_{t+i}} (N(t+10) - N(t))$ , where  $N(t)$  and  $N(t+10)$  are the

housing stocks measured during the two closest censuses. Thus, the change in housing stock is portioned across years based on the observed permitting activity.

We then estimate the time series properties of income shocks at the local level, by fitting an ARMA(1,1) to an income series that is first demeaned with city and year fixed effects and then corrected for city size changes. Using the adjusted-BEA personal income measure, this procedure yields estimates of  $\delta=0.87$ ,  $\theta=0.17$ , and  $\sigma_\varepsilon^2 = \$3.6$  million, which are reported in Table 1.<sup>26</sup>

We then perform the analogous adjustments to the HMDA-based income series and re-estimate the ARMA (1,1).<sup>27</sup> For the post-1990 period, when we have both HMDA and BEA data, we find that the variance of income shocks ( $\sigma_\varepsilon^2$ ) in the HMDA data is roughly double (\$5.7 million) the variance of income shocks found using the BEA data (\$2.8 million).<sup>28</sup>

We also examined whether volatility is higher in high demand markets, and we use proximity to the coast as a proxy for demand. With the BEA data, we re-estimated  $\sigma_\varepsilon^2$  for a sample of 31 markets whose centers are within 50 miles of the Atlantic or Pacific Oceans. While the AR ( $\delta$ ) and MA ( $\theta$ ) components were little changed from those reported for the 115 market national sample, the estimate of  $\sigma_\varepsilon^2$  is almost 50 percent higher in the 31 coastal markets: \$5.3 million versus \$3.6 million. Not surprisingly, income volatility is even higher for recent buyers in these same markets. The  $\sigma_\varepsilon^2$  estimate from the HMDA series is \$7.8 million.<sup>29</sup>

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<sup>26</sup> Largely because  $\gamma$  is so small throughout its relevant range, this adjustment to wages does not have a material impact on our results. If we use a value of 0 for  $\gamma$ , we estimate a value of  $\delta = 0.86$ , an estimate of  $\theta = 0.18$ , and an estimate of  $\sigma_\varepsilon^2 = \$3,408,250$ . In addition, we attempted joint maximum likelihood estimation of  $\delta$ ,  $\theta$ , and  $\sigma_\varepsilon^2$  for given trend effects and metropolitan area fixed effects, but the program would not converge because the panel was too short relative to the number of markets.

<sup>27</sup> This series uses the median income for each year and MSA of those who were approved for a mortgage for the stated purpose of buying a home.

<sup>28</sup> These results suggest that income volatility was relatively high in the 1980s. Shocks also appear to have been less permanent since then. The estimate of  $\delta$  is 0.67 in both series (compared to 0.87 for the BEA series since 1980). The moving average component is weaker, too, as  $\theta=0.08$  using the HMDA data and  $\theta=0.14$  in the BEA data since 1990.

<sup>29</sup> The difference in volatility of local wage shocks across different types of markets is large, and to our knowledge, has neither been well-documented nor well understood. Highly productive coastal areas might

These estimates provide a plausible range for  $\sigma_e^2$ . For the nation as a whole, the figures run from a low of \$3.6 million based on the BEA series to \$5.7 million using the HMDA data. We also experiment with the higher income shock volatility of \$7.8 million found in the high demand, coastal markets using the HMDA series. Results are presented using all three of these values, but the \$3.6 million value is used in our baseline estimation. To keep the presentation simpler, we use the values of  $\delta=0.87$  and  $\theta=0.17$  values from the BEA data, since these parameters do not differ as much from data set to data set.

#### IV. Calibrating the Model and Matching the Data: Baseline Results

In this section, we calibrate the model using the following parameters values from Table 1:  $r=0.04$ ,  $\alpha=0.1$ ,  $\delta=0.87$ ,  $\theta=0.17$ ,  $\delta=0.87$ , and  $\sigma_e^2=\$3.6$  million. Simulation results using these parameter values are reported for seven different values of  $c_1$  (\$0.15, \$0.50, \$2.00, \$5.00, \$10.00, \$20.00, \$50.00) and three values of  $c_2$  (0,  $0.25*c_1$ , and  $0.50*c_1$ ). Hence, there are 21 simulation results for each feature of the data we try to match. Our “real data” sample is a set of 115 metropolitan areas for which we have continuously defined price data from 1980-2005.

We first analyze how sensitive certain model predictions are to small changes in the underlying information structure. Those results lead us to focus on the serial correlation properties and variances of prices and construction. The results in Table 2 document why we use these moments and not some others to see whether the model is consistent with the data.

Our baseline information structure assumes that people learn about shocks during the period (a year in our case) they occur. The first two columns in Table 2 report our model’s predictions for a number of variables presuming such contemporaneous knowledge. To simplify the presentation,  $c_1$  is allowed to take on two values, 2 or 10. Thus, the numbers in the first row of the top panel indicate that the predicted mean reversion coefficient on construction ranges from 0.55 to 0.61 using annual data. The next two rows of that panel report predicted mean reversion over three and five year

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specialize in idea-intensive industries that themselves are relatively volatile, but that is an issue for future research.

intervals.<sup>30</sup> The next two columns report results assuming people learn about shocks one period (year) ahead. We are particularly interested in whether predicted values are sensitive to this change because we do not believe it is possible to pin down the timing of when information is known so precisely. The results indicate that the serial correlation properties of construction intensity basically are unaffected by the change in the underlying information structure.

The next panel in the table shows much the same is true with respect to the predicted variance of construction activity. The third and fourth panels report the analogous predictions for the mean reversion and variance of house price changes. Over annual horizons, the model predicts very slight mean reversion with no foresight of shocks, and slight persistence with knowledge one year ahead. However, this change is relative to a small base number and the impact of different information at the longer three- and five-year horizons is even more modest. There also is not a major impact on the predicted variance of price changes as the numbers in the fourth panel indicate.

However, this stability in predicted values does not hold for correlations with price changes. The fifth panel of Table 2 reports the model's predictions of the correlation between house price and income changes. When people learn about income shocks only as they occur, the correlation between incomes changes and price changes at one year intervals almost is perfect. However, if people know about the income shock a year ahead of time, the correlations drop to 0.10. The next panel shows that correlations between price changes and construction are similarly sensitive. Very strong positive correlations of 0.70-0.76 when contemporaneous knowledge is assumed fall to about zero for one year horizons. These simulations cast serious doubt on the value of any examination of the correlations between high frequency price changes and other variables.

We also looked at the relationship between income changes and construction. The final panel of Table 2 shows that this relationship is more stable than the ones involving price changes, but the predicted correlations change by 0.2 to 0.4 points across

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<sup>30</sup> For any  $j$  year interval, these predictions reflect the relationship between what happened between time  $t$  and  $t-j$  and what happened between time  $t$  and  $t+j$ .

the different underlying information structures depending upon the horizon being investigated.

Given the absence of stability regarding the latter three relationships reported in Table 2, we focus our analysis on the serial correlation properties and volatility of price changes and construction activity. Any inability of our model to accurately match these moments at high frequencies in the data is highly unlikely to be explained by some unobserved difference in the underlying information structure.<sup>31</sup> Hence, we now turn to the analysis of whether the predicted moments match the data, assuming contemporaneous information acquisition.

*Short-Term Momentum and Longer-Term Mean Reversion in Prices, Rents and Permits*

The top row of Table 3 provides evidence on momentum and mean reversion in OFHEO house prices within market over time. We use raw price changes rather than changes in the logarithm of prices in order to be compatible with the model, but our empirical results are not sensitive to such changes in functional form. Since the OFHEO index only provides price increases relative to a base year, we convert this into an implied price series by using the median housing value in the metropolitan area in 1980 as a base price in the metropolitan area and then scale that value by the appreciation in the OFHEO index each year.<sup>32</sup>

The results are estimates from a regression of the current change in prices on the lag change in prices

$$(4) \text{ Price}_{t+j} - \text{Price}_t = \alpha_{MSA} + \gamma_{Year} + \beta(\text{Price}_t - \text{Price}_{t-j}),$$

for j equal to one, three and five years. Because fixed effects estimates such as these which remove market-specific averages can be biased (with spurious mean reversion produced especially when the number of time periods is relatively low), in the first row of

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<sup>31</sup> Over longer horizons, a one-year shift in when information becomes known is less important, so it certainly can make good sense to explore various longer-run relationships with price changes. Because our interest is in higher frequency changes, we do not do that here.

<sup>32</sup> This procedure essentially provides the real price for a constant quality house with the quality being that associated with the median house in 1980. We have experimented with using values from the 1990 and 2000 censuses as the base. All the results reported below are robust to such changes.

Table 3, we report Arellano-Bond estimates which use lagged values of the dependent variable (price changes) as instruments.<sup>33</sup>

Our one year estimate of price change serial correlation is 0.71, implying that a \$1 increase in housing prices between time  $t$  and  $t+1$  is associated with a 71 cent increase between time  $t+1$  and  $t+2$ . Smoothing of the underlying data series will create a bias towards finding short-run momentum. Case and Shiller (1989) address this problem by splitting their sample of individual sales transactions in four markets. They report coefficients ranging from 0.2-0.5, but since they use the logarithm of prices, not the level, the results are not exactly comparable. Since we cannot perform any similar procedure with the OFHEO data, our estimate is surely biased upwards.<sup>34</sup> Over three years, there is still momentum. The estimate of 0.27 means that a \$1 increase in housing prices between time  $t$  and  $t+3$  is associated with a 27 cent increase between time  $t+3$  and  $t+6$ .

Over five year periods, we estimate a mean reversion coefficient of -0.32, so a \$1 increase between times  $t$  and  $t+5$  is associated with a 32 cent decline between time  $t+5$  and  $t+10$ .<sup>35</sup> The mean reversion in prices that we estimate over five-year horizons is quite similar in magnitude to that observed for financial assets by Fama and French (1988).<sup>36</sup> Unfortunately, the short time period for which we have constant quality price data at less than decadal frequencies makes it difficult to know whether this mean

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<sup>33</sup> See Arellano and Bond (1991) for more detail on this estimation procedure. More specifically, we use the “xtabond” Stata command with year and area fixed effects.

<sup>34</sup> The OFHEO index includes data on repeat sales or refinancings of the same house. The latter typically rely on an appraisal, not a market sale price. Undoubtedly, this results in smoothing of the series and biases upward our estimate of short-run momentum. Even the Case and Shiller (1989) estimates, which rely only on actual sales, could be upward biased. Working with a split sample, bias can result if, randomly, some fraction of homes on which a buyer and seller agree on a price have delayed closings that move their reported sales dates into the next reported period. Whatever shock there was in period  $t$  that influenced the agreed upon price, some of its measured impact will spill over into period  $t+1$ . Obviously, this is potentially more of a problem the shorter the measurement period.

<sup>35</sup> These estimates are not an artifact of the Arellano-Bond procedure. The analogous ordinary least squares estimates over 1, 3, and 5 year horizons are 0.74, 0.18 and -0.39, respectively. We also addressed concerns about spurious mean reversion by estimating specifications without metropolitan area fixed effects. If we estimate the following equation,  $Price_{t+5} - Price_t = \alpha + \gamma_{Year} + \beta(Price_t - Price_{t-5})$ , the mean reversion coefficient drops to -0.11 and becomes only marginally significant. However, as soon as we include percent of adults with college degrees as a control, the coefficient becomes -0.18 with a t-statistic of three. If we estimate the same change regression using the logarithm of prices instead of the levels, the coefficient is -0.20 (-0.22 with the college graduate control) and has a t-statistic of four.

<sup>36</sup> Cutler, Poterba and Summers (1991) also find this pattern of short run momentum and longer- run mean reversion for housing and a number of other asset markets.

reversion is a permanent feature of urban life or whether it represents the impact of shocks that are specific to the post-1980 time period.

Table 4 reports the model's predictions for the serial correlation in prices based on simulations using the 21 different  $c_1$  and  $c_2$  combinations discussed above. The first three columns show results for annual serial correlation in prices, the next three columns present the analogous findings over three year periods, with the final three columns being for five year periods.

The one-year predictions document the model's failure to match the strong positive serial correlation observed in the annual data. Our calibration predicts mean reversion even at such a high frequency. The predicted mean reversion is much higher in low cost, more elastic supply areas than in high cost, inelastic supply areas because more new construction will cause housing prices to fall more rapidly in the first group of markets. The results for three year horizons reported in the middle columns of Table 4 also find a mismatch with the data. Assuming the middle case for omega ( $\omega=0.25$ , column 5), we predict mean reversion coefficients from -0.17 to -0.45, not the positive persistence we see in the data as reported in Table 3.

Our model does a much better job of fitting the -0.32 mean reversion seen at five year intervals (columns 7-9). At five year horizons, if  $c_1$  takes on a value from 2 to 5 and  $\omega=0.25$ , then we come within ten percent of matching the data (see rows 3 and 4, column 8). In fact, almost all of the construction cost parameter values predict levels of mean reversion that are close to those observed in the data. Only in areas with extremely elastic supply do we predict mean reversion that substantially differs from observed levels.

The model generates mean reversion both through the tendency of shocks to mean revert and of new construction to cause future declines in prices. New construction will only decrease future housing prices when housing demand is downward sloping, so when demand for housing is perfectly elastic (i.e.,  $\alpha=0$ ), the only force for mean reversion is the mean reversion of shocks. When we restrict  $\alpha$  to equal 0, the predicted baseline level of mean reversion is about -0.25, so only a small amount of mean reversion in the average market is due to the effects of new construction. By contrast, if we set  $\delta$  to equal 1, so that there is no mean reversion in the shocks, then the predicted mean reversion

disappears almost entirely, especially in markets with high values of  $c_1$  and  $c_2$ . In those markets, the mean reversion of shocks, not new construction activity, seems to drive the mean reversion of prices. In markets with lower values of  $c_1$  and  $c_2$ , new construction plays a more important role in generating the mean reversion of prices.<sup>37</sup>

Short run momentum in asset price changes is thought by some to provide evidence of anomalies in the asset markets. If this momentum reflected some asset market quirk, then presumably it should not also appear in rents. In the second row of Table 3, we report the results from rental regressions of the form in equation (5),

$$(5) \text{ Rent}_{t+j} - \text{Rent}_t = \alpha_{MSA} + \gamma_{Year} + \beta(\text{Rent}_t - \text{Rent}_{t-j}).$$

Rental data on apartments is collected by an industry consultant and data provider, REIS Inc. Their data covers only a limited number of metropolitan areas (46 in our sample), and in general, rental units are not similar to owner-occupied housing.<sup>38</sup>

Over one- and three-year horizons, there is strong evidence of persistence, with the Arellano-Bond estimates being 0.27 in both cases. Over five year time horizons, we estimate a mean reversion parameter of -0.64. The presence of high frequency momentum and low frequency mean reversion in rents suggests that these features do not reflect something unique to housing asset markets, but rather something about the changing demand for cities.<sup>39</sup>

Table 5 then reports the predicted values of serial correlation in rents from the simulations of the model. At annual frequencies, we predict serial correlation ranging from -0.26 to 0.09 when  $\omega=0.25$  (column 2). Even though there is no predicted mean reversion in higher cost, more inelastic markets, these estimates still are well below the 0.27 estimate observed in the data. Over three year horizons, we consistently predict mean reversion, while there still is a positive serial correlation of rents in the data. For five year intervals, we predict that rent changes should have a mean reversion coefficient of between -0.31 and -0.37 if we assume that  $c_1$  lies between 2 and 5 and  $\omega=0.25$  (row 3

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<sup>37</sup> In these more elastic markets, if  $\alpha=0$ , then predicted mean reversion over five year periods falls by a quarter relative to the numbers in Table 4.

<sup>38</sup> Rental units are overwhelmingly in multi-unit buildings, while owner-occupied housing is overwhelmingly single-family detached housing. These differences in housing types and the problem of accurately measuring maintenance costs are two reasons why it is extremely difficult to tell whether housing prices are high or low relative to rents.

<sup>39</sup> The ordinary least squares estimates of these coefficients are 0.28, 0.08 and -0.51 for one, three and five year horizons, respectively.

and 4, column 8). Higher mean reversion is predicted if  $c_1$  is lower, but our estimates still are only about one-half of the observed mean reversion in that case.

Thus, we are again unable to explain the strong positive serial correlations at shorter time horizons. However, because of the many reasons to be suspicious about the properties of the rental data, especially because of artificial smoothing, we attach less importance to the quantitative mismatch with the data here.<sup>40</sup>

To examine the dynamics of housing quantities, we turn to housing permit data from the *Census of Construction*. The final set of results reported in Table 2 use housing permits estimated in the following regression:  $Permits_t^{t+j} = \alpha_{MSA} + \gamma_{Year} + \beta Permits_{t-j}^t$ , where  $Permits_{t-j}^t$  refers to the number of permits issued between time  $t-j$  and time  $t$ . The one-three and five year Arellano-Bond coefficient estimates are 0.84, 0.43, and -0.07, respectively. Thus, construction also displays high frequency momentum, but little or no persistence or mean reversion at longer horizons.<sup>41</sup>

The calibration results for this variable are provided in Table 6. For the case where  $c_I=5$  and  $\omega=0.25$ , the predicted coefficients are 0.60, 0.28 and 0.05, for one, three, and five year horizons, respectively. These are moderately close to the actual parameters, and minor changes in the values of one or both of the supply side parameters enable us to fit the data more exactly. While the predictions about the serial correlation of construction are not as accurate as the predictions about the mean reversion in prices, the moments of the real data cannot be said to reject the model.

In sum, the model does a reasonable job at fitting the time series properties of new building and an excellent job at fitting the long term mean reversion of prices. It does a very poor job of fitting the high frequency positive serial correlation of price changes. This failure may be the result of data smoothing causing us to empirically overestimate momentum, or as Case and Shiller (1989) suggest, it could reflect some sort of irrationality in the housing market. However, the predictable mean reversion of prices at lower frequency five year intervals is not a challenge for a rational expectations model.

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<sup>40</sup> For example, smoothing is a greater problem in the rental data. The industry consultant that provides the rent data does not survey actual renters, but the landlord owners of apartment buildings. Undoubtedly, averages are being reported.

<sup>41</sup> As is the case with the other data, this pattern is not an artifact of our estimation procedure. The analogous ordinary least squares coefficients are 0.82, 0.37, and 0.07, respectively.

### *House Price Change and Construction Variances Across Markets*

Table 7 reports the variance of price changes and of new construction in our sample.<sup>42</sup> The volatility of both prices and construction varies enormously across cities. The distribution is quite skewed, with the mean variance much higher than the median variance. To address this heterogeneity, we first run a regression for each outcome (price change or permits) using all of our markets controlling for year effects, and then compute the variance of the residuals from this regression by metropolitan area. This variance gives us the volatility of prices and construction, respectively, within a metropolitan area controlling for nationwide effects.

The top panel of Table 7 shows that that the variance of one year price changes computed this way equals \$14 million in the metropolitan area from the 10<sup>th</sup> percentile of the distribution and \$209 million in the market at the 90<sup>th</sup> percentile of the distribution. The median market has a one year price change variance of \$34 million, which is much smaller than the sample mean of \$83 million. This skewness is driven primarily by California markets and Honolulu.<sup>43</sup> The second and third columns of this top panel of Table 7 report the distribution of variances of three and five year price changes. The distribution of longer horizon price changes is also quite skewed, with the mean price change substantially exceeding the change for the median area. The variance in five-year price changes is \$625 million for the median market, and one quarter of the metropolitan areas have variances of at least \$1.1 billion.

Table 8 reports predicted price change variances from our simulations with the results arrayed in the same manner as in the serial correlation tables above. At annual frequencies (columns 1-3), we predict a range of price change variances from a low of \$16 million to a high of \$190 million. Not surprisingly, price change volatility is lower for smaller values of  $c_1$  and  $c_2$ . In those markets, quantities can respond readily to changes in demand. The actual \$34 million price variance of the median market lies within this large range of predicted values, but it is only compatible with a very low value

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<sup>42</sup> Since the rent data are smoothed, we do not believe much, if any, weight should be put on the measured variance of rents. Hence, that variable is excluded from this part of the analysis.

<sup>43</sup> The variance of one-year price changes in Honolulu is \$763 million, which is the largest in our sample. Five other markets—San Jose, San Francisco, Santa Barbara, Santa Ana and Salinas--had variances that were at least ten times greater than the sample median.

of  $c_I : 0.50$ . For more typical values of  $c_I$ , we predict much higher variation in one year price changes than we see in the data.

Data smoothing would bias measured price volatility downward over short time periods, and if so, we would expect this problem to be less severe for longer time periods. Our ability to match the volatility of price changes in the median market does increase with the horizon over which those changes are measured. If  $c_I=2$  (and  $\omega=0.25$ ), the predicted price variance over three-year horizons is \$182 million, which is quite close to the \$185 variance found in the median market. If  $c_I=0.5$  (and  $\omega=0.25$ ), the predicted variance almost exactly matches that found in the 10<sup>th</sup> percentile market in the data. However, at this longer horizon, the model fails to match the high price volatility seen in the top quartile of housing markets. Even assuming a very inelastic supply side (i.e.,  $c_I=\$20$  or  $\$50$ ), we do not predict a house price change variance much above the \$445 million observed in the 75<sup>th</sup> percentile market.

For the 5-year price change variance predictions listed in the final three columns of Table 8, our range of predictions runs from \$29 to \$756 million. This captures the lower half of the distribution of actual price change variation reported in the third column of Table 7 (top panel), but our model generally predicts too little price change volatility at this lower frequency. For example, if  $c_I$  equals 5 and  $\omega=0.25$ , then we predict a five year price variance of \$421 million, which still is well below the sample median (\$625 million). Higher  $c_I$  values, of course, allow us to come closer, but no reasonable values of  $c_I$  predict the very high price change volatility found in the top quarter of markets. That the model overestimates price volatility at high frequencies, but not at lower frequencies, might reflect artificial smoothing of the price change data.

Turning to construction, the bottom panel of Table 7 reports the variance in units permitted across our 115 metropolitan area sample. As with price changes, there is substantial heterogeneity in the volatility of construction intensity across markets, and this distribution is skewed by a few outliers. For example, the bottom quartile of markets has a new construction variance of about 2 million units per annum, while the top quartile is at least five times more volatile. Moreover, this distribution is skewed by relatively few markets in the right tail that have variances of at least 38 million units.

Table 9 then reports the construction variance estimates from our standard set of simulations. Our model essentially can predict almost any construction variation given the full range of construction costs estimates. At annual periods, the range of actual variances, which run from two million in the 10<sup>th</sup> percentile market to 38 million at the 90<sup>th</sup> percentile of the distribution, lies within the range of values predicted when  $c_1$  ranges from 50 cents to ten dollars and  $\omega=0.25$ . The median market in our sample has a one year standard deviation of 3 million which is in the 2-5 million unit range predicted when  $c_1$  lies between 5 and 10 and  $\omega=0.25$ .

Over three year horizons, the data also roughly fit the model. For example, the three-year construction variance in the median market is 26 million units (middle column, bottom panel of Table 7). The range of predicted variances is between 11 and 30 million if  $c_1$  lies between 5 and 10 and  $\omega=0.25$ . The most volatile markets are also compatible with the model, if  $c_1$  is sufficiently low. For example, the variance of 328 million observed for the 90<sup>th</sup> percentile market, is close to the variance predicted for a market where  $c_1$  equals 0.5 and  $\omega=0.25$ .

At five year intervals, the actual median market has a variance of 59 million units, which is quite close to the variance of 62 million predicted when  $c_1$  equals 5 and  $\omega=0.25$ . The 10<sup>th</sup>-90<sup>th</sup> percentile range in the data lies between 29-760 million units. When  $\omega=0.25$ , the predicted range is between 22 and 986 million when  $c_1$  ranges from 0.15 to 10. Overall, the construction variances in the data are well within the range of variances predicted by the data. The values of  $c_1$  and  $c_2$  must be quite low to explain the places with extremely volatile construction levels, but we find believe that is plausible for a number of American housing markets.

Thus, supply elasticity can explain a significant amount of the high construction volatility in the Sunbelt, but supply inelasticity cannot explain nearly as much of the high price volatility in coastal markets. We now ask whether extensions to the model can explain the high volatility of house price changes in those markets.

## **V. Explaining the High Volatility of Housing Prices**

The simplest explanation of highly volatile housing prices are omitted demand shocks, such as changes to local tax rates and amenities, and mismeasurement of income

shocks. We examine those hypotheses first and then turn to the potential role of time-varying real interest rates.

### *Local Tax Rates and Amenities*

Changes in local tax rates and amenity flows could increase housing volatility. We use data from the NBER TAXSIM website on the average tax rate on wage income earned in a given state each year to create an after-tax income measure for each market. Our analysis of after-tax income showed that controlling for this factor cannot be responsible for more than a 10 percent increase in local demand variability which would translate into a ten percent increase in price and construction volatility. While Appendix II provides the details, we conclude that changes in state level tax rates cannot be driving the high price volatility in the top quartile of markets.

Unlike taxes, most amenities are relatively permanent characteristics of a place, (e.g., the weather, local architecture). The demand for these amenities may change slowly over time as a society becomes richer or more unequal or as new technologies become available, but it is hard to imagine that their value will fluctuate much at annual frequencies. Crime represents one of the few amenities that does change relatively rapidly and for which there is available data. We collected violent crime rates for the largest cities in each of our metropolitan areas using continuous crime data from 1985-2005.<sup>44</sup> We then created an adjusted income variable that subtracted the negative effect of crime from our BEA real income measure. As detailed in Appendix III, the results showed almost no impact on the variability of local demand from controlling for crime. We infer from this that we are unlikely to find an amenity with high frequency variation that can explain much of the observed volatility in prices or construction.

### *Measurement of Income Shocks*

Because the key outstanding price-related puzzle for our model is the high volatility of lower frequency price changes in the top quartile of metropolitan areas, the top panel of Table 10 provides new estimated variances of five-year price changes for

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<sup>44</sup> We emphasize that this measure is for the local political jurisdiction, which we then impute to the metropolitan area.

markets with high  $c_1$  values of at least 10, assuming more variable local demand shocks. We use the two estimates of  $\sigma_\varepsilon^2$  from the HMDA data discussed above, \$5.6 million and \$7.8 million, to reflect the plausible range of higher income shock volatilities, with the first column reproducing the baseline estimates from column 8 of Table 8 for comparison purposes.

The first row of the second column of Table 10, which assumes a value of \$5.7 million for  $\sigma_\varepsilon^2$ , indicates that one still needs an extremely high  $(c_1, c_2)$  pair to match the price change volatility of the 75<sup>th</sup> percentile market. However, if  $\sigma_\varepsilon^2$  is doubled as is assumed in the third column and which seems plausible for the coastal markets based on the HMDA data, predicted price change volatility comfortably reaches the \$1.17 billion level observed in the data for the 75<sup>th</sup> percentile market (see the final column of the top panel in Table 10).

This still leaves the model unable to account for the very high price change variances seen in the top ten percent of markets, which are all on the Pacific coast. The 12 most volatile markets in our sample comprise Honolulu and much of coastal California.<sup>45</sup> There are no east coast markets in this group, with the Nassau-Suffolk and Bethesda-Gaithersburg-Frederick metropolitan areas having the 15<sup>th</sup> and 16<sup>th</sup> biggest price change volatilities.

The bottom panel of Table 10 shows that the combination of elastic supply side conditions and higher local income shock variation allows the model to match most of the highest construction intensity variation markets in the country. We allow demand side volatility to increase only to \$5.7 million in these simulations because there are no coastal markets with such low  $c_1$  and  $c_2$  values. In this case, we no longer need extremely low values for the supply side parameters to account for the construction intensity variation in the most volatile markets. Given that we would expect the volatility of the marginal buyer's income to be greater than the average used in the baseline simulations, these results suggest that high construction volatility is not a puzzle.

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<sup>45</sup> The top ten percent of the most volatile metropolitan areas in terms of five-year price changes (in ascending order from #104-#115) are as follows: Oakland-Fremont-Hayward, Santa Cruz-Watsonville, San Luis Obispo-Paso Robles, San Diego-Carlsbad-San Marcos, Oxnard-Thousand Oaks-Ventura, Los Angeles-Long Beach-Glendale, San Jose-Sunnyvale-Santa Clara, Salinas, Santa Ana-Anaheim-Irvine, San Francisco-San Mateo-Redwood City, Santa Barbara-Santa Maria, and Honolulu.

### Time-Varying Interest Rates

We have so far assumed that interest rates are fixed for reasons of tractability, but we recognize that many authors have claimed that the dramatic rise in house prices, especially in high cost markets, over the past decade is best understood as a response to declining interest rates that make housing in those areas more affordable (e.g., Himmelberg, Mayer and Sinai, 2005). A full treatment of interest rates would require an analysis of long period mortgages and prepayment that lies well beyond the scope of this paper. We can, however, adjust the model modestly to acquire some understanding of the potential impact of time-varying interest rates. In doing so, we assume a spot market for refinancing, as well as linearity in the difference equation describing the shocks to rates.

We begin by decomposing interest rates into permanent and transitory components,  $\bar{r}$  and  $\rho(t)$ , respectively, where  $r(t) = \bar{r} + \rho(t)$ , and use the approximations

$$\frac{1}{1+r(t)} \approx \frac{1}{1+\bar{r}} \text{ and } r(t)(H(t) - C) = \bar{r}(H(t) - C) + \rho(t)(\bar{H} - C), \text{ where } \bar{H} \text{ is meant to}$$

reflect the average housing price in the city.<sup>46</sup> If we adjust equation (2) for time-varying interest rates using these approximations, equation (2') results:

$$(2') \quad H(t) - \frac{\bar{r}C}{1+\bar{r}} - \frac{E_t(H(t+1))}{1+\bar{r}} = \bar{D} + qt + x(t) - \rho(t)(\bar{H} - C) - \alpha N(t)$$

If we then make the admittedly unrealistic assumption that  $\rho(t) = \lambda\rho(t-1) + \eta(t)$  so the difference equation remains linear, the model can be solved in a relatively straightforward fashion.

This results in equation (5)'s description of prices,

$$(5) \quad H(t) = \hat{H}(t) + \frac{(c_1\phi + \Psi)x(t)}{c_1(\phi - \delta) + \Psi} + \frac{c_1\theta\varepsilon(t)}{c_1(\phi - \delta) + \Psi} - \frac{\alpha(1+r)}{1+r-\phi} (N(t) - \hat{N}(t)) - \frac{(c_1\phi + \Psi)(\bar{H} - C)\rho(t)}{c_1(\phi - \lambda) + \Psi}$$

<sup>46</sup> The first approximation is minor and would have been unnecessary if we assumed that the utility flow was received at the end of the period rather than the beginning of each period. The second approximation eliminates interactions between transitory changes in value and transitory changes in the interest rate and it may be more consequential.

This differs from the price equation in Proposition 1 because of its last term which multiplies  $\frac{c_1\phi + \Psi}{c_1(\phi - \delta) + \Psi}$  times the interest rate shock times  $\bar{H} - C$  (the gap between average housing prices in the area and construction costs). This term reflects the fact that a decline in interest rates essentially is a positive demand shock for high amenity and productivity places. The shock makes it cheaper to live in such places, pushing up demand and prices.

Since our regressions correct for year effects, this interest rate effect can have no impact on our estimates for the average market which will have prices close to construction costs. The interest rate effect does, however, have the capacity to generate increased variance in both price changes and construction levels for places that are considerably more expensive on average. We consider four different values of  $(\bar{H} - C)$ : \$25,000, \$50,000, \$100,000 and \$200,000, which over the past 25 years captures most of the range of American metropolitan areas.<sup>47</sup>

We set  $c_1$  and  $c_2$  equal to 3.5 and 0.875, respectively, in order to focus on interest rate effects.<sup>48</sup> We assume that  $\lambda=0.90$ , but experimentation with values as high as 0.95 yield similar results. The variance of interest rates is far more important for the results, and we use a range of standard deviations for  $\eta(t)$  from 0.005 to 0.02.<sup>49</sup>

Simulations suggest that these changes to our baseline model make little difference to the amount of predicted mean reversion. Thus, the results in Table 11 focus on the impact of interest rate volatility on the variance of price changes. In general, the results from this table show that interest rates shocks can generate significant increases in the variance of price changes if the volatility of rates is at least one percent and if the market has house prices much greater than the average. If  $H-C < \$100,000$ , the impact on predicted price change variation is minimal. For markets in which  $H-C = \$200,000$ , the

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<sup>47</sup> For example, in 1980 the highest price metropolitan area had a median house value that was about \$170,000 greater than in the median market. The real value of median market's median house price is barely changed between 1980 and 2000. Except for a handful of markets in the upper tail of the metropolitan area price distributions, gaps in excess of \$200,000 with the median market do not exist.

<sup>48</sup> These represent the mid-point between  $c_1=2$  and  $c_1=5$  (assuming  $\omega=0.25$ ), which we believe reflect typical supply side conditions.

<sup>49</sup> Campbell's (2000) review of the asset pricing literature notes that the standard deviation on a one period riskless asset is 1.76 percent, but concludes that "... perhaps half ... is due to ex post inflation shocks (p. 1519)." Thus, the lower half of this range seems more plausible. Recent asset pricing papers such as Bansal, Kiku, and Yaron (2006) assume a standard deviation of 1 percent.

impact typically is quite large. Over high frequency, annual periods, a predicted variance almost equaling the \$209 million figure observed for the 90th percentile metropolitan area (see Table 7) results if  $H-C=\$200,000$  and if the standard deviation of  $\eta(t)$  is one percent. This is roughly double the predicted variance reported in Table 8 for the same supply side conditions (i.e.,  $c_1=3.5$ ;  $c_2=0.875$ ). Many of the coastal California markets in question have median house values that are \$200,000 greater than the national median, and assuming a standard deviation of one percent for the one period rate is not extreme. Moreover, if we run the same simulation, but assume relatively inelastic supply side parameter values (i.e.,  $c_1=10$ ;  $c_2=2.5$ ), the predicted price change variance increases to \$307.

While this indicates that interest rate shocks may be able to help account for the highest price change volatility observed at high frequencies, the same cannot be said for longer horizons. Recall from Table 7 that the observed price change variance in the 90<sup>th</sup> percentile market is \$1.38 billion for three-year horizons. Table 11 indicates that to even approach that number not only requires very high house prices, but extremely high short rate volatility. Specifically, if  $H-C=\$200,000$  and the standard deviation of  $\eta=0.02$ , the predicted price change variance is \$1.29 billion.<sup>50</sup> A similar pattern is observed at five year intervals, except the underprediction of price change volatility is even more extreme.<sup>51</sup>

Table 12 shows that interest rate shocks do not have much influence on predicted variation in construction intensity. Generally speaking, unless both  $\bar{H} - C$  equals \$200,000 and the standard deviation of interest rate shocks is 0.02, the predicted construction volatilities are within the range of values reported in our baseline simulations for markets with  $c_1$  values between 2 and 5 (see Table 8).

In sum, these results are consistent with the claims of Himmelberg, Mayer, and Sinai (2005) and others that time-varying interest rates can play a role in helping explain house price behavior in certain high value markets. However, these findings do not

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<sup>50</sup> Even assuming more inelastic supply with  $c_1=10$  and  $c_2=2.5$ , a more plausible assumption of  $\eta=0.01$  yields a predicted price change variance of only \$802, which is still only 58 percent of the observed volatility in the 90<sup>th</sup> percentile market.

<sup>51</sup> For example, if we assume a very high interest rate volatility ( $\eta=0.02$ ) and inelastic supply ( $c_1=10$ ;  $c_2=2.5$ ), the predicted price change variance is \$3.01 billion—still 16 percent below the \$3.58 billion observed in the 90<sup>th</sup> percentile market.

indicate that rate variation does or can explain the extremely high price change volatility observed in certain west coast markets over multiple year periods. Further work will hopefully ground our understanding of interest rates and housing prices with a formal model of long-term mortgages with prepayment. Without such a model, our results should be viewed only as a rough heuristic guide on the matter of time-varying rates.

## **VI. Conclusion**

This paper presents a dynamic rational explanations model of housing markets based on a cross-city spatial equilibrium. The model predicts that housing markets will be largely local, which they are, and that construction persistence is fully compatible with price mean reversion. The model is also consistent with price changes being predictable.

The model has successes and failures at fitting the real data. The model can explain the serial correlation of construction quantities reasonably well and can explain the five year mean reversion of prices almost perfectly. However, the model cannot explain the high frequency positive serial correlation of price changes. The model can explain the price and construction volatility of a typical housing market. It does a good job of accounting for the heterogeneity in construction intensity variation across most markets in the country. However, it cannot account for the most volatile markets in terms of low frequency price changes, which are located along California's coast.

This suggests that housing economists begin to focus their attention on high price volatility in coastal markets and on the positive serial correlation of high frequency price changes. The average volatility and longer-term mean reversion of prices should no longer be viewed as puzzles. In addition, the time series properties of new construction and the volatility of changes in building activity are well understood by a dynamic, rational equilibrium model that allows for differences in supply elasticity across markets.

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<b>Table 1: Model Parameters</b>	
r	0.04
$\delta$	0.87
$\theta$	0.17
$\sigma_e^2$	BEA: \$3.6 million; HMDA: \$5.7 million; HMDA (coastal markets): \$7.8 million
$\alpha$	0.1
$c_1$	\$0.15, \$0.50, \$2.00, \$5.00, \$10.00, \$20.00, \$50.00 (per housing unit)
$c_2$	$c_2$ can be 0%, 25%, or 50% of $c_1$ ; hence, there are 21 different combinations of ( $c_1, c_2$ ) pairs

<b>Table 2: Sensitivity to Predictions of Different Information Structures</b>				
	Contemporaneous Knowledge		Knowledge One Year Ahead	
	$c_1=2$	$c_1=10$	$c_1=2$	$c_1=10$
<b>Predicted Mean Reversion of Construction</b>				
1 year	0.55	0.61	0.58	0.63
3 years	0.23	0.31	0.23	0.61
5 years	0.00	0.07	0.00	0.07
<b>Predicted Variance of Construction (millions of units)</b>				
1 year	17	2	21	2
3 years	95	11	112	9
5 years	189	22	245	29
<b>Predicted Mean Reversion of Price Changes</b>				
1 year	-0.10	-0.06	0.11	0.10
3 years	-0.27	-0.19	-0.21	-0.15
5 years	-0.34	-0.28	-0.32	-0.26
<b>Predicted Variance of Price Changes (\$millions)</b>				
1 year	76	146	54	127
3 years	182	382	175	413
5 years	245	550	249	622
<b>Predicted Correlation of Price Changes with Income Changes</b>				
1 year	0.97	0.99	0.10	0.10
3 years	0.96	0.99	0.62	0.67
5 years	0.96	0.99	0.73	0.77
<b>Predicted Correlation of Price Changes with Construction</b>				
1 year	0.70	0.76	-0.05	0.03
3 years	0.48	0.59	0.29	0.32
5 years	0.52	0.61	0.44	0.47
<b>Predicted Correlation of Income Changes with Construction</b>				
1 year	0.57	0.64	0.81	0.76
3 years	0.46	0.48	0.82	0.74
5 years	0.59	0.52	0.86	0.80

Note:  $c_2=0.25*c_1$ . All other parameters as defined in Table 1 ( $r=0.04$ ,  $\delta=0.87$ ,  $\theta=0.17$ ,  $\alpha=0.1$ ,  $\sigma_\varepsilon^2=\$3.6$  million).

<b>Table 3: Variation in Prices and Quantities Within-Market Over Time</b> Arellano-Bond Estimates of Coefficients on Lagged Dependent Variable 1, 3, & 5 year horizons			
<i>Dependent Variable</i>	<i>1-year changes</i>	<i>3-year changes</i>	<i>5-year changes</i>
House Price Change	0.71 (0.01) N=2,819	0.27 (0.04) N=690	-0.32 (0.07) N=345
Rent Change	0.27 (0.03) N=1,007	0.27 (0.08) N=274	-0.64 (0.17) N=91
New Permits	0.84 (0.01) N=2,645	0.43 (0.04) N=690	-0.07 (0.06) N=460

Notes:

1. Sample for house price, employment, and permit specifications is 115 metropolitan area sample described in text.
2. Sample for rent specification is 46 metropolitan areas tracked by REIS.

**Table 4: Predicted Mean Reversion of Prices**

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$
$c_1 = .15$	-0.24	-0.27	-0.29	-0.46	-0.45	-0.40	-0.51	-0.47	-0.41
$c_1 = .50$	-0.15	-0.18	-0.18	-0.39	-0.37	-0.30	-0.48	-0.42	-0.34
$c_1 = 2$	-0.08	-0.10	-0.09	-0.28	-0.27	-0.22	-0.39	-0.34	-0.28
$c_1 = 5$	-0.06	-0.07	-0.06	-0.23	-0.22	-0.19	-0.34	-0.30	-0.26
$c_1 = 10$	-0.05	-0.06	-0.05	-0.20	-0.19	-0.18	-0.30	-0.28	-0.25
$c_1 = 20$	-0.04	-0.05	-0.05	-0.18	-0.18	-0.17	-0.28	-0.26	-0.25
$c_1 = 50$	-0.04	-0.05	-0.04	-0.17	-0.17	-0.17	-0.26	-0.25	-0.25

**Table 5: Predicted Mean Reversion of Rents**

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$
$c_1 = .15$	-0.21	-0.26	-0.32	-0.45	-0.46	-0.47	-0.50	-0.49	-0.48
$c_1 = .50$	-0.08	-0.14	-0.21	-0.36	-0.39	-0.40	-0.46	-0.45	-0.43
$c_1 = 2$	0.03	-0.02	-0.07	-0.24	-0.28	-0.27	-0.36	-0.37	-0.33
$c_1 = 5$	0.06	0.03	0.01	-0.19	-0.22	-0.20	-0.30	-0.31	-0.28
$c_1 = 10$	0.08	0.06	0.05	-0.16	-0.18	-0.17	-0.27	-0.28	-0.26
$c_1 = 20$	0.09	0.08	0.07	-0.15	-0.16	-0.15	-0.25	-0.26	-0.24
$c_1 = 50$	0.09	0.09	0.09	-0.14	-0.14	-0.14	-0.24	-0.24	-0.24

**Table 6: Predicted Mean Reversion of Construction**

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$
$c_1 = .15$	0.36	0.28	0.18	0.04	-0.02	-0.08	-0.14	-0.17	-0.20
$c_1 = .50$	0.54	0.43	0.29	0.22	0.10	-0.01	-0.01	-0.10	-0.17
$c_1 = 2$	0.70	0.55	0.36	0.43	0.23	0.04	0.20	-0.00	-0.14
$c_1 = 5$	0.76	0.60	0.39	0.54	0.28	0.06	0.33	0.05	-0.13
$c_1 = 10$	0.80	0.61	0.39	0.60	0.31	0.07	0.42	0.07	-0.12
$c_1 = 20$	0.82	0.62	0.40	0.65	0.32	0.07	0.48	0.08	-0.12
$c_1 = 50$	0.84	0.63	0.40	0.69	0.33	0.07	0.55	0.09	-0.12

<b>Table 7: Variance in House Price Changes and Construction Intensity 1, 3, and 5 Year Horizons</b>			
	<i>House Price Change Variance (millions of \$2000)</i>		
	1 year	3 years	5 years
10 <sup>th</sup> percentile market	\$14	\$69	\$183
25 <sup>th</sup> percentile market	\$26	\$124	\$452
50 <sup>th</sup> percentile market	\$34	\$185	\$625
75 <sup>th</sup> percentile market	\$70	\$445	\$1,170
90 <sup>th</sup> percentile market	\$209	\$1,380	\$3,580
Sample mean	\$83	\$484	\$1,310
	<i>Construction Intensity Variance (millions of units)</i>		
	1 year	3 years	5 years
10 <sup>th</sup> percentile market	2	13	29
25 <sup>th</sup> percentile market	2	19	41
50 <sup>th</sup> percentile market	3	26	59
75 <sup>th</sup> percentile market	11	84	212
90 <sup>th</sup> percentile market	38	328	760
Sample mean	21	160	417

**Table 8: Predicted Variance of Prices, Millions**

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$
$c_1 = .15$	16	16	18	27	27	28	29	29	32
$c_1 = .50$	28	33	40	59	67	82	70	80	105
$c_1 = 2$	54	76	97	134	182	239	180	245	337
$c_1 = 5$	80	117	140	209	298	365	293	421	528
$c_1 = 10$	102	146	164	272	382	436	391	550	637
$c_1 = 20$	125	167	179	336	446	481	489	650	708
$c_1 = 50$	151	184	190	410	496	513	604	728	756

**Table 9: Predicted Variance of Construction, Millions**

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$
$c_1 = .15$	117	141	172	538	587	634	946	986	1016
$c_1 = .50$	48	62	80	275	313	337	544	575	569
$c_1 = 2$	13	17	19	87	95	87	191	189	153
$c_1 = 5$	4	5	5	32	30	23	74	62	41
$c_1 = 10$	2	2	1	13	11	7	31	22	13
$c_1 = 20$	1	1	0.4	5	3	2	12	7	4
$c_1 = 50$	0.2	0.1	0.1	1	1	0.3	3	1	1

**Table 10: The Impact of Greater Local Demand Variability**

<b>Table 10: The Impact of Greater Local Demand Variability</b>				
<i>Five-year Price Change Variance (\$millions)</i>				
	Baseline $\sigma_\varepsilon^2$ (= \$3.6)	$\sigma_\varepsilon^2 = \$5.7$	$\sigma_\varepsilon^2 = \$7.8$	
$c_1=10; c_2=2.5$	550	871	1,192	
$c_1=20; c_2=5$	650	1,029	1,408	
$c_1=50; c_2=12.5$	728	1,153	1,577	
<i>Five-Year Quantity Change Variance (millions of units)</i>				
	Baseline $\sigma_\varepsilon^2$ (= \$3.6)	$\sigma_\varepsilon^2 = \$5.7$		
$c_1=0.15; c_2=0.0375$	986	1,561		
$c_1=0.50; c_2=0.1250$	575	910		
$c_1=2; c_2=0.75$	189	299		

<b>Table 11: Predicted Variance of Price Changes (\$millions): Interest Rate Volatility 1, 3, and 5 Year Horizons <math>c_1=3.5, c_2=0.875</math>; all other parameter values as reported in Table 1.</b>									
	One-Year			Three-Year			Five-Year		
$\bar{H} - C$	Stand. Dev. ( $\eta$ ) = 0.005	Stand. Dev. ( $\eta$ ) = 0.01	Stand. Dev. ( $\eta$ ) = 0.02	Stand. Dev. ( $\eta$ ) = 0.005	Stand. Dev. ( $\eta$ ) = 0.01	Stand. Dev. ( $\eta$ ) = 0.02	Stand. Dev. ( $\eta$ ) = 0.005	Stand. Dev. ( $\eta$ ) = 0.01	Stand. Dev. ( $\eta$ ) = 0.02
\$25,000	100	102	107	253	256	268	350	355	372
\$50,000	102	107	127	256	268	317	355	372	441
\$100,000	107	127	206	268	317	511	372	441	714
\$200,000	127	206	522	317	511	1288	441	714	1808

Note: All parameter values are as reported in Table 1, expect that  $c_1=3.5$  and  $c_2=0.875$ .

<b>Table 12: Predicted Variance of Construction (millions of units): Interest Rate Volatility 1, 3, and 5 Year Horizons</b>									
	One-Year			Three-Year			Five-Year		
$\bar{H} - C$	Stand. Dev. ( $\eta$ ) = 0.005	Stand. Dev. ( $\eta$ ) = 0.01	Stand. Dev. ( $\eta$ ) = 0.02	Stand. Dev. ( $\eta$ ) = 0.005	Stand. Dev. ( $\eta$ ) = 0.01	Stand. Dev. ( $\eta$ ) = 0.02	Stand. Dev. ( $\eta$ ) = 0.005	Stand. Dev. ( $\eta$ ) = 0.01	Stand. Dev. ( $\eta$ ) = 0.02
\$25,000	8	8	9	49	50	53	100	101	107
\$50,000	8	9	11	50	53	63	102	107	128
\$100,000	9	11	17	53	63	104	107	128	214
\$200,000	11	17	44	63	104	270	128	214	558

Note: All parameter values are as reported in Table 1, expect that  $c_1=3.5$  and  $c_2=0.875$ .



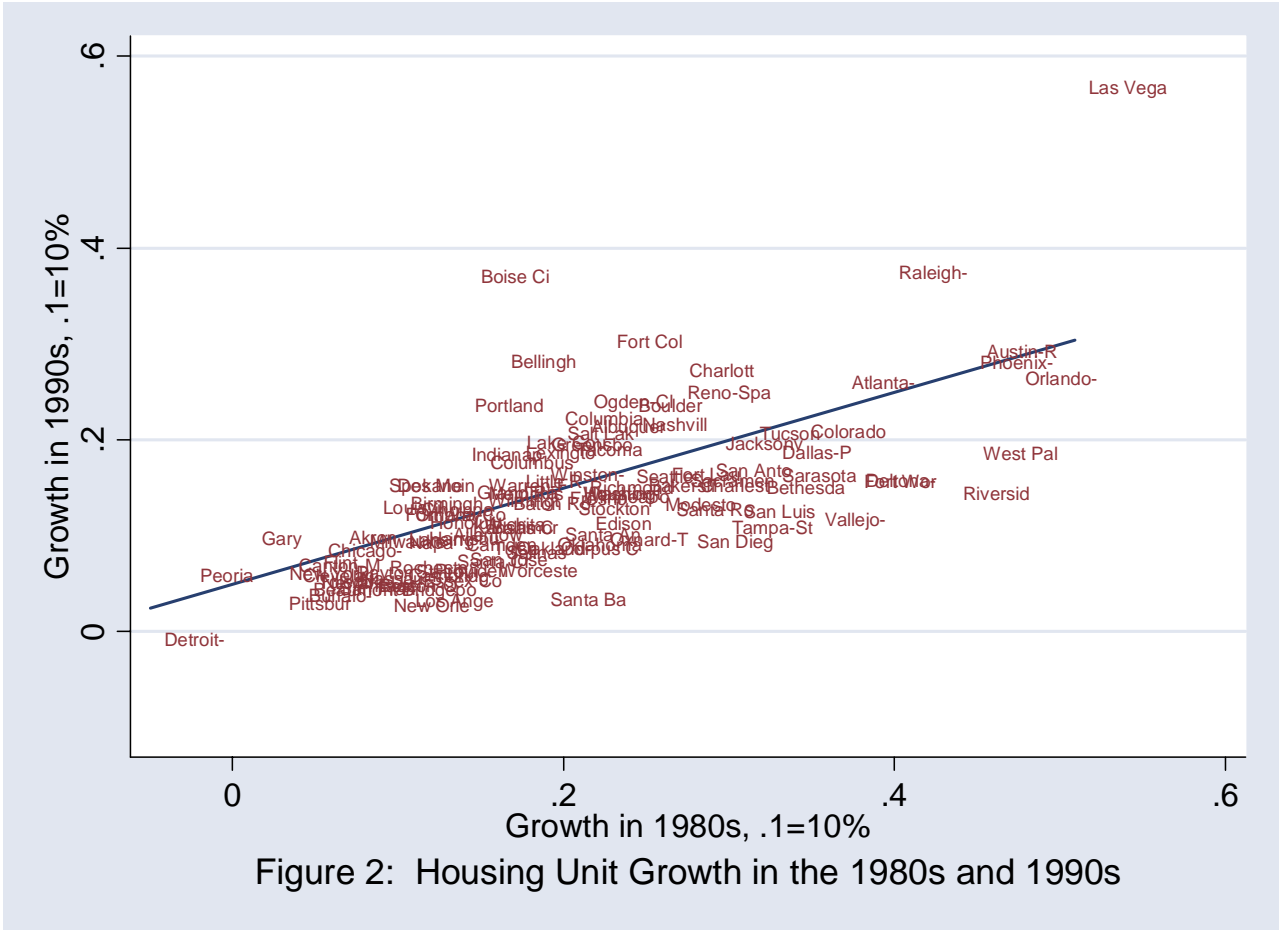
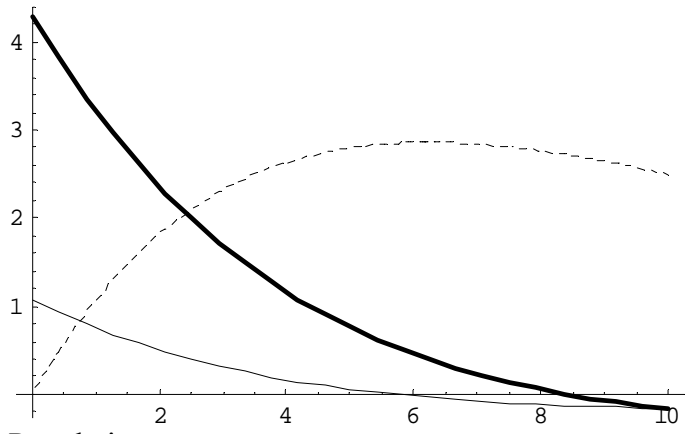


Figure 2: Housing Unit Growth in the 1980s and 1990s

Figure 3: One-Time Shock



Population: -----  
Construction: \_\_\_\_\_  
Price: \_\_\_\_\_

( $\alpha = 0.1, c_1 = 3, c_2 = 0.1$ )

## Appendix 1: Steady State Values and Proofs of Propositions

The steady state values for housing, investment and population are

$$\hat{H}(t) = \frac{\left( \begin{array}{l} c_2^2(q(1+r) + r(rC + \bar{D}(1+r))) + \\ \alpha(1+r)(C + c_1(q(1+r) - rc_0)) + \\ \alpha c_2(1+r)((c_0 + \bar{D} - q)(1+r) + 2rC) \end{array} \right)}{(rc_2 + \alpha(1+r))^2} + \frac{(1+r)(\alpha c_0 + qc_2)}{rc_2 + \alpha(1+r)} t,$$

$$\hat{I}(t) = \hat{I} = \frac{q(1+r) - rc_0}{rc_2 + \alpha(1+r)} \quad \text{and} \quad \hat{N}(t) = \frac{\left( \begin{array}{l} rc_1(rc_0 - q(1+r)) + \alpha(1+r)(c_0 + \bar{D}(1+r)) \\ + c_2(q(1+r)^2 + r(\bar{D}(1+r) - rc_0)) \end{array} \right)}{(rc_2 + \alpha(1+r))^2} + \hat{I}t.$$

*Proof of Proposition 1:* We use the change of variables  $I(t) = m(t) + \hat{I}(t)$ ,

$N(t) = n(t) + \hat{N}(t)$ , and  $H(t) = z(t) + \hat{H}(t)$ . Substituting in our definitions of  $\hat{I}$ ,  $\hat{N}$ , and  $\hat{H}$ , we reduce the core pricing equation

$$H(t) = \bar{D} + qt + x(t) - \alpha N(t) + \frac{rC}{1+r} + \frac{E_t(H(t+1))}{1+r} \quad \text{to}$$

$$z(t) = x(t) - \alpha n(t) + \frac{E_t(z(t+1))}{1+r}, \quad (*)$$

the optimality condition for production  $C + c_0(t+1) + c_1I(t+1) + c_2N(t) = E_t(H(t+1))$  to

$$c_1m(t+1) + c_2n(t) = E_t(z(t+1)), \quad (**)$$

and the defining equation  $I(t+1) = N(t+1) - N(t)$  to

$$m(t+1) = n(t+1) - n(t). \quad (***)$$

We seek functions  $n$ ,  $z$ , and  $m$  that satisfy the starred equations.

Define  $u \equiv \frac{\alpha}{c_1}$  and  $v \equiv \frac{c_2}{c_1}$ ;  $0 \leq u$  and  $0 \leq v < 1$  by the conventions in force. Then

$$\phi = \frac{1}{2} \left( 2+r + (1+r)u - v - \sqrt{r^2 + v^2 + 2(1+r)(2+r)u + (1+r)^2u^2 + 2v(r - (1+r)u)} \right)$$

and

$$\bar{\phi} = \frac{1}{2} \left( 2+r + (1+r)u - v + \sqrt{r^2 + v^2 + 2(1+r)(2+r)u + (1+r)^2u^2 + 2v(r - (1+r)u)} \right).$$

Because  $0 \leq v < 1$ , the expression under the radical is positive. Note that

$$\phi + \bar{\phi} = 2+r + (1+r)u - v > (1+r)(1+u) > 0 \quad \text{and} \quad \phi\bar{\phi} = (1+r)(1-v) > 0, \quad \text{so} \quad \phi, \bar{\phi} > 0.$$

Also,  $\sqrt{r^2 + v^2 + 2(1+r)(2+r)u + (1+r)^2u^2 + 2v(r - (1+r)u)} >$

$$\sqrt{2(1+r)(2+r)u + (1+r)^2u^2 - 2(1+r)u} > \sqrt{2(1+r)(1+r)u + (1+r)^2u^2} > 1+r,$$

so

$$\bar{\phi} > \frac{1}{2}(1+r+(1+r)u+1+r) \geq 1+r > 1,$$

which in turn gives

$$0 \leq \phi = \frac{(1+r)(1-v)}{\bar{\phi}} < 1-v \leq 1.$$

Now define  $n$  by the difference equation

$$n(t) - \phi n(t-1) = \frac{1+r}{c_1(\bar{\phi}-\delta)} E_{t-1}(x(t)). \quad (1)$$

A unique solution  $n$  exists because  $\bar{\phi} > 1 > \delta$  ensures  $\bar{\phi} - \delta \neq 0$  and because  $|\phi| < 1$  allows us to solve for  $n$  explicitly as

$$n(t) = \frac{1+r}{c_1(\bar{\phi}-\delta)} \sum_{i=0}^{\infty} \phi^i L^i E_{t-1}(x(t)) \quad (2),$$

where  $L$  denotes the lag operator. Now that we have defined  $n$ , we set

$$z(t) \equiv x(t) + \frac{1}{\bar{\phi}-\delta} E_t(x(t+1)) - \frac{\alpha(1+r)}{1+r-\phi} n(t) \quad (3)$$

and

$$m(t+1) \equiv \frac{1+r}{c_1(\bar{\phi}-\delta)} E_t(x(t+1)) - (1-\phi)n(t). \quad (4)$$

With these choices for  $z$  and  $m$ , (\*) reduces to

$$\frac{\alpha}{1+r-\phi} (n(t+1) - \phi n(t)) = \frac{\bar{\phi}-1-r}{(\bar{\phi}-\delta)(1+r)} E_t(x(t+1)), \quad (5)$$

which by (1) is equivalent to

$$\frac{u(1+r)}{(1+r-\phi)(\bar{\phi}-\delta)} E_t(x(t+1)) = \frac{\bar{\phi}-1-r}{(\bar{\phi}-\delta)(1+r)} E_t(x(t+1)),$$

which is true, as one sees from cross-multiplying the coefficients and using the previously established formulas for the product and sum of  $\phi$  and  $\bar{\phi}$ . (\*\*\*) reduces to

$$\frac{\alpha(1+r)}{1+r-\phi} \left( n(t+1) - \frac{1+r-\phi}{\alpha(1+r)} (c_1(1-\phi) - c_2)n(t) \right) = \frac{\bar{\phi}-1-r}{\bar{\phi}-\delta} E_t(x(t+1)),$$

which is equivalent to (5), and thus true, because

$$\phi = \frac{1+r-\phi}{\alpha(1+r)} (c_1(1-\phi) - c_2) = \frac{(1+r-\phi)(1-\phi-v)}{u(1+r)},$$

itself evident from cross-multiplying and using the fact that  $\phi$  satisfies the quadratic equation  $y^2 - (2+r+(1+r)u-v)y + (1+r)(1-v) = 0$ . Finally, (\*\*\*) reduces to (1).

This shows that our choices for  $n$ ,  $z$ , and  $m$  solve the starred equations. To recover Proposition 1, we use  $I(t) = m(t) + \hat{I}(t)$ ,  $N(t) = n(t) + \hat{N}(t)$ , and  $H(t) = z(t) + \hat{H}(t)$ .

[The result then follows from  $E_t(x(t+1)) = \delta x(t) + \theta \varepsilon(t)$ .]

*Proof of Proposition 2:* First note that by induction on  $i \geq 1$ ,

$E_t(x(t+i)) = E_t(\delta x(t+i-1) + \theta \varepsilon(t+i-1)) = \delta E_t(x(t+i-1)) = \delta^{i-1} E_t(x(t+1))$ , so from (2),

$$\begin{aligned} E_t(n(t+j)) &= \frac{1+r}{c_1(\bar{\phi}-\delta)} E_t \sum_{i=0}^{\infty} \phi^i L^i E_{t+j-1}(x(t+j)) \\ &= \frac{1+r}{c_1(\bar{\phi}-\delta)} \left( \sum_{i=j}^{\infty} \phi^i L^i E_{t+j-1}(x(t+j)) + \sum_{i=0}^{j-1} \phi^i E_t(x(t+j-i)) \right) \\ &= \frac{1+r}{c_1(\bar{\phi}-\delta)} \left( \phi^j \sum_{i=0}^{\infty} \phi^i L^i E_{t-1}(x(t)) + \sum_{i=0}^{j-1} \phi^i \delta^{j-1-i} E_t(x(t+1)) \right) \\ &= \phi^j n(t) + \frac{1+r}{c_1(\bar{\phi}-\delta)} \frac{\phi^j - \delta^j}{\phi - \delta} E_t(x(t+1)). \quad (6) \end{aligned}$$

Using (4) and (6), we next find that

$$\begin{aligned} E_t(m(t+j)) &= \frac{1+r}{c_1(\bar{\phi}-\delta)} E_t(x(t+j)) - (1-\phi)E_t(n(t+j-1)) \\ &= \frac{1+r}{c_1(\bar{\phi}-\delta)} \left( \delta^{j-1} - (1-\phi) \frac{\phi^{j-1} - \delta^{j-1}}{\phi - \delta} \right) E_t(x(t+1)) - \phi^{j-1} (1-\phi)n(t) \\ &= \frac{1+r}{c_1(\bar{\phi}-\delta)} \left( \frac{\delta^{j-1}(1-\delta) - \phi^{j-1}(1-\phi)}{\phi - \delta} \right) E_t(x(t+1)) - \phi^{j-1} (1-\phi)n(t). \quad (7) \end{aligned}$$

Finally, using (\*\*), (6), and (7), we get

$$\begin{aligned} E_t(z(t+j)) &= c_1 E_t(m(t+j)) + c_2 E_t(n(t+j-1)) \\ &= \frac{1+r}{c_1(\bar{\phi}-\delta)} \left( c_1 \frac{\delta^{j-1}(1-\delta) - \phi^{j-1}(1-\phi)}{\phi - \delta} + c_2 \frac{\phi^{j-1} - \delta^{j-1}}{\phi - \delta} \right) E_t(x(t+1)) \\ &\quad - \phi^{j-1} (c_1(1-\phi) - c_2) n(t) \\ &= \frac{1+r}{\bar{\phi}-\delta} \left( \frac{\delta^{j-1}(1-\nu-\delta) - \phi^{j-1}(1-\nu-\phi)}{\phi - \delta} \right) E_t(x(t+1)) - c_1 \phi^{j-1} (1-\nu-\phi) n(t). \quad (8) \end{aligned}$$

To recover Proposition 2, we use  $I(t) = m(t) + \hat{I}(t)$ ,  $N(t) = n(t) + \hat{N}(t)$ ,

and  $H(t) = z(t) + \hat{H}(t)$  with equations (3), (8), (4), (7), and (6).

*Proof of Proposition 3:* Given the hypotheses, we have

$x(t) = \delta x(t-1) + \theta \varepsilon(t-1) + \varepsilon(t) = \varepsilon(t) > 0$ ,  $E_t x(t+1) = \delta x(t) + \theta \varepsilon(t) = (\delta + \theta) \varepsilon(t) > 0$ , and  $n(t) = 0$ , so from (3) and (4) we deduce that  $z(t) > 0$  and  $m(t+1) > 0$ : prices and investment will initially be higher than steady state levels. By assumption,  $\nu = 0$ , so by (7) and (8), each of expected time  $t+j$  construction,  $E_t(m(t+j))$ , and expected time  $t+j$  price,  $E_t(z(t+j))$ , is negative if and only if

$$\frac{\delta^{j-1}(1-\delta) - \phi^{j-1}(1-\phi)}{\phi - \delta} < 0. \quad (9)$$

If  $\phi > \delta$ , then (9) holds if and only if

$$\frac{1-\delta}{1-\phi} < \left(\frac{\phi}{\delta}\right)^{j-1},$$

which holds for sufficiently large  $j$  because  $\phi/\delta > 1$ . If  $\phi < \delta$ , then (9) holds if and only if

$$\frac{1-\phi}{1-\delta} < \left(\frac{\delta}{\phi}\right)^{j-1},$$

which holds for sufficiently large  $j$  because  $\delta/\phi > 1$ . If  $\phi = \delta$ , then we reduce (9) to

$$0 < \frac{\phi^j - \delta^j - (\phi^{j-1} - \delta^{j-1})}{\phi - \delta} = \frac{(\phi - \delta) \left( \sum_{i=0}^{j-1} \phi^i \delta^{j-1-i} - \sum_{i=0}^{j-2} \phi^i \delta^{j-2-i} \right)}{\phi - \delta} = \phi^{j-2} (j\phi - j + 1),$$

which holds for sufficiently large  $j$  because  $0 \leq \phi < 1$ . This shows that there exists  $j^*$  such that for all  $j > j^*$ , time  $t$  expected values of time  $t + j$  construction and housing prices will lie below steady state levels. When  $\varepsilon(t) < 0$ , we swap  $>$  and  $<$  to recover the symmetric case.

*Proof of Proposition 4:* By assumption,  $n(0) = 0$ , so from (1), we have

$$n(1) = \frac{(1+r)\delta}{c_1(\bar{\phi} - \delta)} \varepsilon(0). \quad (10)$$

From (\*\*\*),  $m(1) = n(1)$ , and from (4) and (10),

$$m(2) = \frac{(1+r)\delta}{c_1(\bar{\phi} - \delta)} ((\delta + \phi - 1)\varepsilon(0) + \varepsilon(1)).$$

By definition,  $I(t) = \frac{q(1+r) - rc_0}{rc_2 + \alpha(1+r)} + m(t)$ , so

$$\text{Cov}(I(2), I(1)) = \left( \frac{1+r}{rc_2 + \alpha(1+r)} \right)^2 \text{Var}(q) + \left( \frac{(1+r)\delta}{c_1(\bar{\phi} - \delta)} \right)^2 (\delta + \phi - 1) \text{Var}(\varepsilon),$$

which is positive if and only if

$$\frac{\text{Var}(q)}{\text{Var}(\varepsilon)} > (1 - \delta - \phi) \left( \frac{\delta(rc_2 + \alpha(1+r))}{c_1(\bar{\phi} - \delta)} \right)^2.$$

From (3) and (10),

$$z(0) = \frac{\bar{\phi}}{\bar{\phi} - \delta} \varepsilon(0) \quad (11)$$

and

$$z(1) = \frac{1}{\bar{\phi} - \delta} \left( \left( \bar{\phi} - \frac{u(1+r)^2}{1+r-\phi} \right) \delta \varepsilon(0) + \bar{\phi} \varepsilon(1) \right). \quad (12)$$

To compute  $z(2)$ , we use (1) and (10) to get

$$n(2) = \frac{(1+r)\delta}{c_1(\bar{\phi} - \delta)} ((\phi + \delta)\varepsilon(0) + \varepsilon(1)),$$

which yields via (3) that

$$z(2) = \frac{1}{\bar{\phi} - \delta} \left( \left( \bar{\phi} \delta - \frac{u(1+r)^2(\phi + \delta)}{1+r-\phi} \right) \delta \varepsilon(0) + \left( \bar{\phi} - \frac{u(1+r)^2}{1+r-\phi} \right) \delta \varepsilon(1) + \bar{\phi} \varepsilon(2) \right). \quad (13)$$

By definition,  $H(t) = \hat{H}(0) + \frac{(1+r)(\alpha c_0 + q c_2)}{r c_2 + \alpha(1+r)} t + z(t)$ , so from (11), (12), and (13),

$$\begin{aligned} \text{Cov}(H(2) - H(1), H(1) - H(0)) &= \left( \frac{(1+r)c_2}{r c_2 + \alpha(1+r)} \right)^2 \text{Var}(q) \\ &\quad - \frac{\left( \frac{u(1+r)^2 \delta}{1+r-\phi} + (1-\delta)\bar{\phi} \right) \left( \frac{u(1+r)^2 \delta}{1+r-\phi} (1-\delta-\phi) + (1-\delta+\delta^2)\bar{\phi} \right)}{(\bar{\phi} - \delta)^2} \text{Var}(\varepsilon), \end{aligned}$$

which is negative if and only if

$$\Omega \left( \frac{r c_2 + \alpha(1+r)}{(1+r)c_1 c_2 (\bar{\phi} - \delta)} \right)^2 > \frac{\text{Var}(q)}{\text{Var}(\varepsilon)}.$$

## Appendix II: The Contribution of Taxes Local Demand Variance

Data on the average tax rate paid each year in each state was matched to our metropolitan areas using files from the NBER's TaxSim web page. We then multiplied our income numbers by one minus the average tax rate, and calculated new values of  $\delta$ ,  $\theta$  and  $\sigma_\varepsilon^2$  for this adjusted after-tax income measure. The new "after-tax" values of the three parameters are very similar to those used in our simulations:  $\delta=0.87$ ,  $\theta=0.18$ , and  $\sigma_\varepsilon^2=\$3.3$  million. The latter is 92 percent of the \$3.6 million figure obtained without any adjustment for taxes. Hence, correcting for taxes creates an eight percent reduction in the variance and almost no change in the other parameters. Consequently, we conclude that including state level tax rates does not offer any hope of explaining the particularly high price change volatilities.

## Appendix III: The Contribution of Crime to Local Demand Variance

We began by drawing on the hedonic literature on the costs of crime. The range of estimates of the elasticity of property value with respect to the violent crime rate run from 0.05 to 0.15.<sup>52</sup> To turn these housing price elasticities into estimates of the impact of crime on the flow of utility measured in dollar units, we multiply the elasticity by the average housing price per crime to obtain a relationship between the price of housing and the level of crime. We then followed our model and multiplied this figure by  $r/(1+r)$  to generate an estimate of the impact of crime on the flow of utility measured in dollars.

Using this method, our elasticity range from 0.05 to 0.15 implies that the impact of violent crime on the flow of well-being ranges from \$35 to \$105. The upper bound estimate of \$105 dollars implies that, if the violent crime rate in a city increases from 12 violent crimes per 1,000 inhabitants (the national mean) to 24 violent crimes per 1,000 inhabitants, then this is equivalent to an income loss of about \$1,260 dollars, which we believe is a reasonable result.

We then used this upper bound impact to adjust the underlying BEA real income variable and  $\delta$ ,  $\theta$  and  $\sigma_\varepsilon^2$ . As with taxes, crime had little impact on the volatility of the local income shock. Specifically, there is only a 1.4 percent greater shock variance when controlling for crime.<sup>53</sup> While the crime data is far from perfect for our purposes, this exercise leads us to conclude that variation in local amenities will explain little of the high variance price change or construction markets.

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<sup>52</sup> See Thaler (1978) for the lower bound estimate and Schwartz, Susin, and Voicu (2003) for the upper bound number.

<sup>53</sup> We were able to obtain crime data for the major cities of 105 of our 115 metropolitan areas. The ARMA estimates of  $\delta$  and  $\theta$  are virtually unchanged depending upon whether income is adjusted for crime in these 105 markets. As noted, the variability of the 'after-crime' income shock is marginally higher.

## Appendix IV: Year and Metropolitan Area Fixed Effects Regression Results

Each specification described below was estimated using the data on real house prices (in \$2000, created from the OFHEO constant quality price index as described in the text) for the 115 metropolitan areas for which we have continuous annual observations from 1980-2005.

1. Price Levels and Year Fixed Effects ( $R^2=0.08$ , nobs=2,990)

$$\text{Price}_{i,t} = \alpha + \beta_t * \text{Year}_t + \varepsilon_{i,t}$$

where  $i$  represents the metropolitan area,  $t$  the year,  $\text{Year}_t$  is a vector of dichotomous year dummies,  $\beta_t$  is the vector of regression coefficients on those year dummies and  $\varepsilon_{i,t}$  is the standard error term.

2. Annual Price Changes and Year Fixed Effects ( $R^2=0.27$ , nobs=2,875)

$$\Delta \text{Price}_{i,t} = \alpha + \beta_t * \text{Year}_t + \varepsilon_{i,t}$$

where  $i$  represents the metropolitan area,  $t$  the year,  $\text{Year}_t$  is a vector of dichotomous year dummies,  $\beta_t$  is the vector of regression coefficients on those year dummies and  $\varepsilon_{i,t}$  is the standard error term.

3. Price Levels and Metropolitan Area Fixed Effects ( $R^2=0.78$ , nobs=2,990)

$$\text{Price}_{i,t} = \alpha + \gamma_i * \text{MSA}_i + \varepsilon_{i,t}$$

where  $i$  represents the metropolitan area,  $t$  the year,  $\text{MSA}_i$  is a vector of dichotomous metropolitan area dummies,  $\gamma_i$  is the vector of regression coefficients on those metropolitan area dummies and  $\varepsilon_{i,t}$  is the standard error term.

4. Price Levels with Year and Metropolitan Area Fixed Effects ( $R^2=0.86$ , nobs=2,990)

$$\text{Price}_{i,t} = \alpha + \beta_t * \text{Year}_t + \gamma_i * \text{MSA}_i + \varepsilon_{i,t}$$

where  $i$  represents the metropolitan area,  $t$  the year,  $\text{Year}_t$  is a vector of dichotomous year dummies,  $\beta_t$  is the vector of regression coefficients on those year dummies,  $\text{MSA}_i$  is a vector of dichotomous metropolitan area dummies,  $\gamma_i$  is the vector of regression coefficients on those metropolitan area dummies, and  $\varepsilon_{i,t}$  is the standard error term.