The Tax Treatment of Housing and Its Effects on Bounded and Unbounded Communities

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I. Introduction

This paper analyzes how tax code-related benefits for housing can influence urban form, with a special focus on their role in financing population decentralization and residential sorting by income. Although it has long been recognized that the favorable tax status of owner-occupied housing has resulted in more housing investment than otherwise would have occurred, housing-related tax policies generally have not been thought to have an important impact on the pattern of development of metropolitan areas.[1] In this paper, we show that the federal tax treatment of owner-occupied housing can have divergent impacts across communities within a metropolitan area. In particular, the effects–both in terms of capitalization of the subsidy and the incentive for households to sort by income--on older, fully-developed communities or "bounded communities" differ widely from the effects on communities with plentiful undeveloped land or "unbounded communities" that typically are located on the urban fringe.

[1]Mills (1986) and others have shown that the U.S. tax treatment of housing has resulted in a substantial overinvestment in housing. Since land is an input into housing production, we would also expect that there is an overinvestment of residential land, resulting in metropolitan areas that have lower population densities. Analyses of the patterns of metropolitan development generally have not, however, seriously considered the impacts of the tax treatment of housing. In their analysis of the causes of metropolitan suburbanization, for example, Mieszkowski & Mills (1993) consider transportation policies and the like, but not housing-related tax expenditures.

The distinction between bounded and unbounded communities is important because supply elasticities play an important role in determining the ultimate impacts of housing-related tax benefits. This is easiest to see in bounded or fully developed areas where the supply of land is relatively inelastic so that the value of any subsidy to ownership largely is capitalized into price. With the after-tax cost of housing essentially unchanged, housing subsidies have little impact on the quantity demanded of residential land. While density is not affected in this situation, housing subsidies do have an impact on sorting because the benefits vary across owners. The value of mortgage and property tax deductions, as well as the non-taxation of the implicit return on home equity, increase with the owner's marginal tax rate. Thus, the subsidies to housing vary positively with income. The mortgage interest and local property tax deductions alone are estimated by Sinai (1997) to yield about $56 billion annual in tax expenditures. While a full accounting requires consideration of the non-taxation of imputed rent, for ease of exposition we use tax expenditures to proxy for all the tax benefits of home ownership.[2]

There is no doubt that those benefits can be very large at the household level, too. For example, a household in the 31 percent tax bracket putting 20 percent down on a $225,000 home with a 1.5 percent effective local property tax rate realizes mortgage interest and property tax

[2]In terms of our theoretical models, the distinction is not important. Because the value of the non-taxation of the return to housing also increases with income/marginal tax rate, the qualitative nature of the impact of tax code-related benefits to housing is unaffected by whether only mortgage interest and property tax deductions are considered.

deductions in excess of the standard deduction in the first year of ownership that are equal to over 1.5 percent of house value. The associated tax benefit of over $3,700 annually (if 80 percent leverage is maintained), effectively lowers the after-tax price of housing services.[3]

On the other hand, less than 40 percent of home owners, however, have sufficient income, house value, and leverage to warrant itemizing in lieu of taking the standard deduction. For those moderate income households that do not benefit from itemization, there are no mortgage interest or local property tax-related benefits. Even though they still benefit from the fact that the return to housing is untaxed, these non-itemizers are more likely to find areas with substantial capitalization unaffordable. Thus, a housing subsidy that varies positively with income provides incentives resulting in lower income households being concentrated in communities consisting predominately of other lower income households. Moreover, if there are land market imperfections such as large-lot zoning, the sorting by income associated with tax-capitalization is magnified because lower-income households may be constrained from adjusting downward their housing consumption in response to the higher market prices in communities with high rates of tax capitalization.

[3]Authors' calculations, the details of which are available upon request.

In unbounded communities--those on the urban fringe or those with plentiful undeveloped land for whatever reason--the supply of land is relatively elastic. Capitalization of any housing-related tax expenditures that lower the cost of owning is limited in these communities.[4] The decline in the after-tax cost of ownership results in increased average lot size and a less dense urban area. In the absence of land market imperfections, the effects on residential sorting by income are limited, but to the extent that the larger lot sizes are reinforced by zoning regulations, low income households may be excluded from these communities even though market prices have not increased.[5]

It should be noted that the models presented in this paper are complementary, not competing, with the sorting process that occurs in the monocentric city framework. In addition, our models show that the tax treatment of housing effectively helps finance individual choices of homogenous communities, which encourages sorting consistent with the predictions of the Tiebout model. Our basic conclusion is that more sorting occurs than would otherwise be the case because of the presence of a tax policy whose benefits are positively correlated with income

[4]It is important to distinguish between national and local taxes in this instance. Our statement is with respect to national taxes. Local taxes well may be fully capitalized because of the competition among suburban communities.

[5]Voith (1998) shows that the U.S. tax treatment of housing may induce these communities to adopt exclusionary zoning policies such as large minimum lot sizes, which may have significant effects on the overall pattern of metropolitan development, including sorting by income.

and house size. Because housing-related tax expenditures are large, with significantly different benefits for richer and poorer owners, the land use and sorting impacts of this policy could be empirically important.

In addition, by focusing on a national tax policy that can play out very differently across different types of communities, this work has potentially important implications for future empirical research on tax subsidy capitalization. That research, which finds from 20 percent to 100 percent capitalization, is based on aggregations of observations from different types of communities that are likely to have differing underlying relative supply and demand elasticities.[6] Our models suggest that such 'averaging' may not accurately reflect actual capitalization in many local land markets. Future empirical work should attempt to provide results that control for the heterogeneity in capitalization across submarkets within a metropolitan area.

In the next section, we examine the metropolitan-area impacts of the tax treatment of housing within the framework of a very simple monocentric city model. While the thrust of the paper's key conclusions can be seen within this familiar model, the bulk of the analytics presented in the following sections are performed on models squarely within the Tiebout tradition in which distance, as represented by access to the urban core, plays no role. Land market imperfections such as large lot zoning which turn out to play an important role are more readily introduced in a Tiebout-type model.

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[6]See Capozza, Green, & Hendershott (1995), Abraham & Hendershott (1992), and Sinai (1998) for examples of three different estimation strategies providing the wide range of results noted in the text.

II. Implications of Housing-Related Tax Expenditures in a Monocentric City Model

To better understand the potential impacts of deductibility on metropolitan density and sorting by income, we begin with the familiar monocentric city framework in which there is a flat featureless plane, production in the center, a fixed number of identical workers, and constant commuting costs to the center. Land is supplied perfectly elastically so that there is no capitalization of the tax expenditures with after-tax land prices falling by the full amount of any tax code-related subsidies to owner-occupied housing. Poterba (1991) estimates the subsidy to user costs to be between 10% and 20% depending upon household characteristics. If we assume a 15% subsidy, a price elasticity of residential land of around -1.0 implies that total residential land usage would increase by 15%, with residential density being 15% lower.[7] The radius of the metropolitan area would increase 7.2 percent.[8] This is a very large impact on metropolitan form and it undoubtedly represents the upper bound impact on aggregate land usage.

The lower bound impact on metropolitan area size is zero. This would result if the supply of land were absolutely fixed (e.g., there is a fixed urban boundary for some reason). In this case full capitalization of the subsidy would occur, with all land prices adjusting upward so that after-

[7]This example assumes all households are identical. The next paragraph takes up the issue of different households--rich (itemizers) and poor (non-itemizers). The elasticity estimate is based on Gyourko and Voith (1998).

[8]Recall that the radius = (area/pi)**.5. In addition, physical house size would also change, but we do not consider that issue here.

tax prices were unchanged. Naturally, there are no land consumption impacts in this scenario.

These very simple, stylized examples suffice to show that supply elasticities can have a major influence on how the tax policy affects the pattern of metropolitan development. The impacts are affected in even more interesting ways when there are two types of owners--rich itemizers and poor non-itemizers. Even absent tax policy and itemization issues, it is well known that the monocentric model will result in segregation of the two groups, with the group having the steepest bid rent function near the center. It is possible, however, for the tax code-related benefits to affect the equilibrium location of each group, regardless of capitalization.

This can be seen by considering the case in which all low income residents desire to have a house with a lot of size q_p but the rich, who are eligible for the housing subsidy, τ, desire $q_r(\tau)$ where $q(\tau)$ is increasing in τ. Standard bid-rent functions for the rich and poor are given by equations (1) and (2),

(1) $R_r(d) = (y_r - c_r d - x_r)/q_r(\tau)$

(2) $R_p(d) = (y_p - c_p d - x_p)/q_p$

where R_i is rent, y_i is income, c_i captures commuting costs, d is distance from the center and x_i the reservation consumption of non-housing goods. Suppose that $q_r = q_p$ if $\tau = 0$ and commuting costs are such that $c_r > c_p$ because the value of time is greater for the rich. Then with no housing subsidy, the slope of the bid rent function for the rich, $c_r/q_r(0)$, is steeper than that for the poor, c_r/q_p. With no subsidies, the rich would reside close to the center and the poor in more distant locations.[9] As subsidies for housing increase for the rich, however, $q_r(\tau)$ increases and the bid-

[9]Of course, if the quantity of land demand by the rich in the absence of the subsidy was large enough relative to that demanded by the poor, the rich would choose the more distant

rent function for the rich flattens. If the subsidies are large enough, then the bid-rent function will become flatter than that of the poor, and they will choose more distant locations. Thus, even in a world with perfectly operating land markets, the existence of income-dependent subsidies for housing can influence the equilibrium distribution of the rich and poor.

Of course, the empirical reality of most metropolitan areas is more complex than the world described by the monocentric urban model. Individual communities are likely to differ considerably in the degree to which a change in housing subsidies is capitalized into house prices. Communities with housing units that have relatively low prices or which contain units providing relatively small flows of housing services are unlikely to be in demand by the higher income residents for whom the housing subsidy is most valuable. These localities probably will not experience much of an increase in land consumption or higher prices because little of the subsidy is incident on these communities.

Other types of communities that are more appealing to higher income residents and that are fully developed (or which have been zoned to prevent additional development) are likely to see a greater fraction of the housing subsidy capitalized into property values. In these areas, tax code-related housing subsidies probably will not increase quantity of land consumed per resident (because its after-tax cost is virtually unchanged), but they can have an important sorting effect. Because the after-tax price of housing is higher to poorer, non-itemizers, these communities will be less attractive to these types of households.

Still, because metropolitan areas generally have land available on the urban fringe that is

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location even without a subsidy.

elastically supplied, developed communities or communities that effectively zone out growth probably will not see the subsidy value fully capitalized. Even developed communities will tend to see at least some increase in the quantity of land demanded when there is an increase in subsidies because they compete with communities on the urban fringe where the housing subsidy is not capitalized into price. In effect, there is not full capitalization even if local community land area is fixed. The largest impact, in terms of increase in quantity demanded, however, lies on the urban fringe, where land is elastically supplied. If there is large lot zoning in communities being developed on the urban fringe, the analysis below shows that subsidies to high income households will increase sorting even if there is little or no impact on market prices.

III. Three Models of Bounded and Unbounded Communities

In the models presented below, a bounded community is one with a fixed stock of land suitable for development. Hence, land is supplied perfectly inelastically in this locale. An unbounded community has a potentially infinite supply of land, so its supply elasticity is infinite. These assumptions are for simplicity alone, and do not affect the qualitative nature of any comparative statics result. In our lexicon, the central city always is the bounded community and the suburb is the unbounded community. Naturally, this need not be the case as the results apply to any communities with low and high supply elasticities. Before getting to the analytics, it is useful to outline characteristics that apply to all three models presented.

Characterization of the Metropolitan Area

There is a single metropolitan area consisting of two jurisdictions indexed by $j = (c,s)$, with c denoting the central city and s denoting the suburban jurisdiction. The central city boundary is exogenously given and cannot be changed so that the central city's land area is fixed

in size. In contrast, unimproved land is assumed to be perfectly elastically supplied in the suburban region. Because suburban land is perfectly elastically supplied, its price is equal to the price of agricultural land plus the value of public amenities, which are available only where agricultural land has been converted to residential use. The two jurisdictions are characterized by their pre-subsidy house prices, r_j, as well as location-specific attributes for each jurisdiction, A_j.

In addition, two types of workers indexed by i=(h,l), with h denoting high skill and l denoting low skill, live in the metropolitan area. Each group is fixed in size, with high-skill workers earning wage w^h and low-skill workers earning wage w^l.[10] The distribution of these two groups across the metropolitan area depends on preferences, equilibrium prices and amenities, and housing subsidies.

Characterization of the Housing Subsidy

The mortgage interest and property tax deductions, along with the untaxed return on home equity, can lower the after-tax price of housing. Focusing on the two deductions for simplicity, the value of these tax expenditures for any individual depends on whether she finds it advantageous to use itemized deductions, on her marginal tax rate, and on her level of housing consumption. Generally, the deduction is of higher value for higher-income individuals. For simplicity, we specify the mortgage interest and property tax deductions as a subsidy, τ^i, defined as the fraction of the price of housing services paid by the government. It is also presumed that the level of the standard deduction and the progressivity of the tax code combine to function so that the housing subsidy is available only to high-skill workers. Thus, $0<\tau^h < 1$, with $\tau^l = 0$.

[10]We assume that the metropolitan area is an integrated labor market and that all locations are equally accessible to employment so that there no commuting-related rent or wage gradients.

Preferences

Individuals consume a market good, x, (whose price is the numeraire) and housing services, h_j. In addition, utility is derived from location-specific amenities, A_j. An individual consumer, k, maximizes utility by choosing residential location and optimal quantities of x and h_j given r_j, τ^i, A_j, and w^i. More formally,

(3) $$\text{Max } U^{ik}(x, h_j; A_j) \quad \text{Subject to:} \quad x + (1 - \tau^i) r_j h_j = w^i.$$

Individuals of a given type are assumed to have identical preferences over x, h_j, and A_j, but they differ in their preferences for city or suburban location. The utility function is defined such that the indirect utility function, V^{ik}, takes the following form

(4)
$$\text{High skill:} \quad V^{hk} = V(r_j(1-\tau^h), w^h; A_j) + \varepsilon_j^k$$
$$\text{Low skill:} \quad V^{lk} = V(r_j, w^l; A_j) + \varepsilon_j^k,$$

where V(.) is the systematic component of utility and ε_j^k is the increment to indirect utility associated with the choice of location j. Note that ε_j^k is normalized such that it represents the incremental utility associated with an individual choosing a suburban location. Specifically, let $\varepsilon^{ik} = \varepsilon_s^k - \varepsilon_c^k$ define the relative idiosyncratic preference for locations c and s.

Location Choice

Because all consumers have identical tastes except for idiosyncratic preferences for city or suburban living, the marginal consumer is defined (separately for rich and poor individuals) by that ε^k, ε^{i*}, satisfying

(5) $$\varepsilon^{i*} = V_s - V_c.$$

More formally, for each worker type the marginal consumer is defined such that

$$\varepsilon^{h*} = V(r_s(1-\tau^h), w^h; A_s) - V(r_c(1-\tau^h), w^h; A_j)$$
$$\varepsilon^{l*} = V(r_s, w^l; A_s) - V(r_c, w^l; A_j). \quad (5')$$

By specifying a density function, Ψ^i, for ε^{i*}, the fraction of rich or poor individuals choosing city residences, n¢, can be determined as a function of r_j and A_j.[11]

One further simplification is to focus on the difference in amenities, $A=A_s-A_c$, rather than the absolute levels of amenities in city and suburbs. Thus, the fractions of rich and poor households choosing to live in the city are given by:

$$n¢ = \Psi^i(r_s, r_c, A), \text{ for } i = h, l. \quad (6)\text{-}(7)$$[12]

[11]Note that the fraction of high- or low-skill people living in the central city or suburban portion of the metropolitan area is not directly affected by τ^h because the housing subsidy applies equally (on a percentage basis) to any given city or suburban housing unit. Increasing wealth is presumed not to change one's intrinsic preference for a city versus suburban location. Thus, when subsidies increase, they do not favor city or suburbs. Similarly, w^i does not affect the population distribution because wages for individuals of a given type within the single metropolitan area labor market are assumed to be the same in both jurisdictions. This is not particularly restrictive because wages are independent of the policy of interest.

[12]Because the signs of the first partials of these functions will be important later, it is helpful to determine them now. Downward-sloping demand implies that the fraction of people choosing a city residence declines as city rents increase, so that $\partial\Psi^i/\partial r_c$ = Install Equation Editor and double-click here to view equation.; similarly $\partial\Psi^i/\partial r_s$ = Install Equation Editor and double-click here to view equation.. Increases in city amenities or a reduction in suburban amenities should increase the number of people who choose city residences. Because A represents suburban amenities relative to city amenities, $\partial\Psi^i/\partial A$ = Install Equation Editor and double-click here to view equation..

Housing Demand

Given an indirect utility function, Roy's identity provides the demand for housing by each individual. Given the choices of jurisdictions, aggregate housing demand is a function of r_j, τ^h, w^h, and n^h_j for high-skill individuals and r_j, w^l and n^l_j for low-skill individuals as shown in equations (8)-(11).

(8)-(9) $H^h_j = H^h(r_j(1- \tau^h), w^h ; n^h_j)$, for j=c,s ;

(10)-(11) $H^l_j = H^l(r_j, w^l ; n^l_j)$, for j=c,s.[13]

Housing Supply

Housing services are assumed to be proportional to developed land. Further, there is no vacant land in the city because of another assumption that all land there is developed.[14] Thus, the total supply of housing services in the city, H_c, is fixed. Consequently, the following constraint applies for the city portion of the metropolitan area

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[13]Once again, the first partials demand will prove of interest for the comparative static analysis below. First, the amount of housing consumed by both high and low skilled workers obviously decreases with increases in the price of housing services, so that $\partial H^i_c/\partial r_c =$ Install Equation Editor and double-click here to view equation. (with the analogous result holding for the suburban region). For high skilled individuals who are able to use the mortgage interest deduction, the subsidy to housing consumption increases their demand for housing in the central city, so that $\partial H^h_c/\partial \tau =$ Install Equation Editor and double-click here to view equation. . Finally, city housing demand is increasing in wages and in the number of people choosing to live in the city, with $\partial H^i_c/\partial w^i =$ Install Equation Editor and double-click here to view equation. and $\partial H^i_c/\partial n^i_c =$ Install Equation Editor and double-click here to view equation. .

[14]When population falls in the city in this model, the remaining city residents consume more city housing services. Implicitly, we are assuming that the housing stock adjusts in terms of size of housing to match demand. This is obviously unrealistic in the short run, as housing that

(12) $H_c = H^h_c + H^l_c$

IV. Comparative Statics

Case 1: Fixed Amenities and Wages, No Lot-Size Constraints

The first case considered is one in which wages and amenities are exogenously given and no land-use constraints of any type are present. With w^i and A_j exogenously fixed, equations (6), (7), (8), (10), and (12) form a system of five equations in five variables r_c, n^h_c, n^l_c, H^h_c, H^l_c. Note that r_s does not adjust in this case because agricultural land is perfectly elastically supplied and amenities are fixed by assumption.

To examine the effects of changing the mortgage interest deduction on location choices and housing consumption by high- and low-skill workers, as well as city land prices, these equations can be totally differentiated as follows,

(13) $dn^h_c = \Psi^h_{r_c}\ dr_c$

(14) $dn^l_c = \Psi^l_{r_c}\ dr_c$

(15) $$d\,H^h_c = H^h_{c_t}\,dt + H^h_{c_{r_c}} dr_c + H^h_{c_{n^h_c}} d\,n^h_c$$

(16) $$d\,H^l_c = H^l_{c_{r_c}} dr_c + H^l_{c_{n^l_c}}\ d\,n^l_c$$

(17) $$dH^h = -\,dH^l.$$

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does not match consumer demands often is left vacant. It is noteworthy that this sort of fixity of city housing stock only reinforces the results below associated with sorting by income.

Equations (13)-(17) then can be solved for $dr_c/d\tau$, $dn_c^h/d\tau$, $dn_c^l/d\tau$, $dH_c^h/d\tau$, and $dH_c^l/d\tau$. [Hereafter, we drop the superscript on τ, since it is assumed relevant only for high-skill workers.] Consider first the effects of a change in subsidies on the price of housing services in the city which is given by equation (18),

$$(18)\ \frac{dr_c}{dt} = - \frac{H^h_{c_t}}{H^l_{c_{r_c}} + H^l_{c_{n_c^l}}\Psi^l_{r_c} + H^h_{c_{r_c}} + H^h_{c_{n_c^h}}\Psi^h_{r_c}} > 0.$$

The numerator is positive since the demand for housing increases with the level of the housing subsidy. With respect to the denominator, because the demand for housing by each skill type falls as price increases, the first and third terms are negative. The second and fourth terms are also negative because housing demand is increasing in the number of people choosing the city, but the number choosing the city is decreasing in city prices. Thus, equation (18) is strictly positive. This is not surprising since an increase in housing subsidies increases the overall demand for housing, which, in turn, increases city prices because housing in this part of the metropolitan area is inelasticly supplied.

Solving for the effect of housing subsidies on the distribution of high- and low- skill people yields equations (19) and (20),

$$(19)\text{-}(20)\ \frac{dn_c^i}{dt} = \Psi^i_{r_c}\frac{dr_c}{dt} < 0 \qquad i = h,l.$$

Recall that $\Psi^i_{r_c} < 0$ because fewer people choose to live in the city as city rents increase. And, equation (18) just showed that increases in the federal tax subsidy raise city rents. Thus, a higher housing subsidy reduces the number of high- and low-skill workers choosing to live in the city. Essentially, the housing subsidy causes everyone to substitute housing for other goods. Because

city land is in fixed supply, the rising city prices cause both skill types to shift demand to the suburbs where housing is elastically supplied.[15]

However, because the housing subsidies are usable only by high-skill workers, they have differential effects on housing consumption across worker types. For high-skill workers, the effect on city housing demand is given by equation (21)

$$(21)\ \frac{dH_c^l}{dt} = H^l_{c_{r_c}} \frac{dr_c}{dt} + H^l_{c_{n_c^l}} \frac{dn_c^l}{dt} < 0.$$

Since the amount of city housing purchased by a low-skill person falls with increases in price (i.e., $\partial H_c^l/\partial r_c<0$), and increases with the number of low-skill people in the city (i.e., $\partial H_c^l/\partial n_c^l>0$), both terms of equation (21) are negative. Some low-skill workers end up in the suburbs because the housing subsidy to high-skill workers is driving up city rents. This leaves aggregate demand by the low skill types lower in the city.

For high-skill workers, the housing subsidy has the opposite effect. From equation (12),

$$(22)\ \frac{dH_c^h}{dt} = -\frac{dH_c^l}{dt} > 0.$$

[15]Here we have ignored income effects associated with the differences in wages between high and low skill workers and the increased wealth for higher income households receiving the housing subsidy. In particular, differences in the relative income elasticities for amenities, housing, and the numeraire good could lead to differential rates of exit to the suburbs. The analysis here looks at income compensated choices so that we can focus on the pure relative price effect. We follow the same strategy in Cases 2 and 3. If open space is a luxery good as many suspect, modeling the income effect would only reinforce our results.

Even though the housing subsidy reduces the number of high-skill people choosing to live in the city, housing consumption by the remaining high-skill workers increases. This occurs because the after-subsidy price of housing in the city falls for this group (even though the market price of city housing rises). For this type, fewer people consume more housing in the city.

In summary, introducing a subsidy to ownership that is positively correlated with income increases population decentralization within the metropolitan area and results in a less dense central city/bounded community. Abstracting from income effects (see footnote 14), the policy induces no sorting beyond what would result in its absence. In fact, high-skill workers receiving the subsidy end up consuming more of the bounded community's housing stock as a result of the tax-code-related housing subsidies. As the next model shows, a land market imperfection in the form of a large-lot zoning constraint is needed to change this.

Case 2: Fixed Amenities and Wages, With Lot-Size Constraints

The second case introduces a common suburban land-use restriction in the form of a minimum lot-size requirement for residential development. To help simplify the analysis here, it is assumed that lot-size constraints exist such that no low-skill people choose to live in the suburbs, but the constraints are not binding for high-skill workers.[16] In other words, high-skill workers earn sufficiently high wages that they always choose lots at least as large as the constraint whenever they choose a suburban site. Low-skill workers, on the other hand, have sufficiently low wages that they never choose to purchase a lot as large as the minimum in the suburbs.

[16]The model would generate the same qualitative results with a weaker assumption. The

only requirement is that zoning preclude some low skill workers from choosing a suburban location.

These simplifying assumptions concerning lot size imply the following modifications to the comparative statics analyzed in Case 1. Equation (14) is no longer relevant, since low-skill workers never choose to live in the suburbs, and equation (16) simplifies to:

(23) $d\,H_c^l = H_{c_{r_c}}^l\, dr_c$.

Although the signs of the comparative statics are unchanged from those of Case 1, the effects of a housing subsidy on city rents are greater as shown in equation (24)

(24) $$\frac{dr_c}{dt} = -\frac{H_{c_t}^h}{H_{c_{r_c}}^l + H_{c_{r_c}}^h + H_{c_{n_c^h}}^h \Psi_{r_c}^h} > -\frac{H_{c_t}^h}{H_{c_{r_c}}^l + H_{c_{n_c^l}}^l \Psi_{r_c}^l + H_{c_{r_c}}^h + H_{c_{n_c^h}}^h \Psi_{r_c}^h} > 0.$$

The right-most expression in equation (24) is simply that from equation (18) for Case 1. The intuition behind why city rents are higher in this case with binding lot size constraints in the suburbs is that because low-skill workers cannot adjust by changing location, the overall demand for city housing drops less. The larger impact on rent, however, means that more high-skill people choose to leave the city than otherwise would have occurred (the comparative static is the same equation as equation (17)).

Thus, in the presence of lot-size constraints, tax expenditures such as those arising from the mortgage interest deduction to higher income people foster the separation of the rich from the poor. Of course, this is not meant to imply that sorting would not occur in the absence of the tax policy, only that it is greater than would be the case otherwise. And, as the next model shows, when local public amenities are made a function of the skill of the population base, large-lot zoning helps finance even more sorting.

Case 3: Endogenous Amenities, Fixed Wages, with Lot-Size Constraints

The amenities provided by a community are likely to be affected by the demographic composition of the community. In the third case, we allow local public amenities to be determined endogenously according to the following rule: amenities in a jurisdiction are an increasing function of the number of high-skill workers residing in the jurisdiction. This assumption is consistent with amenities being normal goods whose demand increases with community income and with the existence of peer group effects in which high-skill workers positively affect the utility of high- and low-skill workers alike.[17]

To examine the effects of endogenously produced amenities, the basic model must be augmented with two additional equations: one that determines the effect of amenities on rent and another that determines the level of amenities. Recall that the price of suburban residential land is simply the value of agricultural land plus the value of amenities. Suburban residential land prices vary with A as in equation (25)

(25) $r_s = r(A)$.

The level of relative amenities depends on the number of high skilled workers choosing to live in the city so that

[17]That such effects exist is suggested by the models of Benabou (1993, 1996) and others.

(26) $A = A(n_c^h)$. Because equations (25) and (26) do not arise explicitly from the maximization problem outlined above, some restrictions are needed to ensure sensible outcomes. In particular, we assume that increases in amenities resulting from a greater concentration of high-skill workers do not raise rents so fast as to more than offset the utility from the additional amenities. In addition, we assume that the effects of a change in the number of high income people choosing the city (nh) on amenities, directly and indirectly (via the impacts of amenities on rents), cannot be greater than the change in nh itself. Mathematically, this can be stated as $0 < \Psi_A A_{n_c^h} + \Psi_{r_s} R_A A_{n_c^h} < 1$.[18]

Totally differentiating equations (6), (8), (10), (12), (25), and (26) results in a system of six equations and six unknowns: r_c, r_s, nc, Hh, Hc, A. The differential equations for housing demand and supply (equations (15)-(17)) are unchanged but equation (13) now must take into account changes in suburban prices and relative amenities as shown in equation (27)

(27) $dn_c^h = \Psi_{r_c}^h \, dr_c + \Psi_{r_s}^h \, dr_c + \Psi_A \, dA.$

In addition, the total differentials for equations (25) and (26) are given by

(28) $dr_s = R_A \, dA$

(29) $dA = A_{n_c^h} \, dn_c^h$.

Equations (15), (16), (17), (27), (28), and (29) can be solved for the effects of changes in housing

[18]The assumption that this sum is less than one assures that the model is not explosive. The implication is that when a high income person moves from the city to the suburbs, she does not create an even greater incentive for the next person with the closest idiosyncratic preference for the city also to move to the suburbs.

subsidies on the number of high-skill people choosing to work in the city, rents in the city, relative suburban/city amenities, and housing consumption in the city by high- and low-skill workers.

Consider first the effects of housing subsidies on the fraction of high-skill workers choosing to live in the city. Algebraic manipulation yields:

$$(30)\ \frac{dn_c^h}{dt} = \frac{\Psi_{r_c} H_{c_t}^h}{(1 - \Psi_{r_s} R_A A_{n_c^h} - \Psi_A A_{n_c^h})(-H_{c_{r_c}}^h - H_{c_{r_c}}^l) - \Psi_{r_c} H_{c_{n_c^h}}^h} < 0$$

The numerator is always negative for reasons discussed in the previous cases. Given the assumptions regarding the relationship among amenities, suburban prices and choice of residential location (i.e., as $\Psi_A A_{n_c^h} \geq \Psi_{r_s} R_A A_{n_c^h}$), the denominator is always positive. Thus, making amenities a function of the per capita wealth of the community does not change the direction of this effect. Rather, endogenizing amenities reinforces the effects of the housing subsidy because suburban communities become more attractive while city communities become less attractive in terms of amenities. Mathematically,

$$(31)\ \frac{dA}{dt} = A_{n_c^i} \frac{dn_c^i}{dt} > 0.$$

Because the relative position of the suburbs and city in regard to amenities is endogenous, the effect of housing subsidies on city prices may be positive or negative. The effect of housing subsidies on city prices is shown in equation (32)

$$(32)\ \frac{dr_c}{dt} = \frac{H_{c_t}^h}{-H_{c_{r_c}}^h - H_{c_{r_c}}^l} + \frac{H_{c_{n_c^h}}^h}{-H_{c_{r_c}}^h - H_{c_{r_c}}^l} \frac{dn_c^h}{dt} \lessgtr 0.$$

In the previous two cases, housing prices in the city unambiguously rose because overall demand for housing both in the city and the suburbs rose. In this case, the increased exodus of high-skill workers from the city induced by the housing subsidies lowers the relative attractiveness of the

city's amenities. Thus, the housing subsidy has countervailing effects on city prices as represented by the two terms in equation (32). If amenities are strongly sensitive to the composition of the population, it is possible that the housing subsidy can cause a decline in residential prices in the city.

V. Discussion and Conclusion

These three cases clearly show that any public policy subsidizing home ownership differentially along income lines leads to increased residential sorting by income, especially when combined with the ability to large lot zone. Essentially, the tax code related to housing increases the amount of sorting for any given level of zoning because it increases the net benefits of sorting--beyond those implied by standard motivations to engage in fiscally exclusionary zoning. The results also suggest that such a policy increases an unbounded community's optimal lot size, although we leave that endogenous zoning issue to another paper.

In addition, predictions derived from the third case are consistent with a number of empirical phenomena common in U.S. metropolitan areas: decentralization accompanied by sorting by income with increased concentrations of the poor in the city, low amenities in the city, and relatively low residential prices in the city. That is, decentralization within the metropolitan area is accompanied by socioeconomic decline and weak city land markets. The more sensitive are amenities are to the make-up of the local population, the greater potential for collapsing city land prices.

Finally, determining the empirical importance of the implications of the third case, in particular, should be an important area for future research. Given the underlying complexity of urban areas, it certainly will prove difficult to isolate the effects of a single tax policy. However,

well-posed simulation models may provide good insights on which to build more difficult econometric models.

References

Abraham, Jesse and Patric Hendershott. "Patterns and Determinants of Metropolitan House Prices, 1997-91." *NBER Working Paper #4196* (October 1992).

Benabou, Roland, "Workings of a City: Location, Education, and Production", *Quarterly Journal of Economics*, Vol. 108, no. 3 (August 1993): 619-52.

_____, "Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance, *American Economic Review*, Vol. 86, no. 3 (June 1996): 584-609

Capozza, Dennis, Richard Green and Patric Hendershott. "Taxes, House Prices, and Consumption." *Mimeo*, (December 1995).

Follain, James and David Ling. "The Federal Tax Subsidy to Housing and the Reduced Value of the Mortgage Interest Deduction", *National Tax Journal*, June 1991, 44 (2), 147-168.

Gyourko, Joseph and Richard Voith, "The Price Elasticity of the Demand for Residential Lotsize: Estimation and Implications of Tax Code-Related Subsidies on Urban Form," University of Pennsylvania: Wharton Real Estate Center, mimeo (1998).

Mieszkowski, Peter and Edwin Mills, "The Causes of Metropolitan Suburbanization," *Journal of Economic Perspectives*, Vol. 7, no. 3 (Summer 1993): 135-147.

Mills, Edwin, "Dividing Up the Investment Pie: Have We Overinvested in Housing?" *Business Review*, Federal Reserve Bank of Philadelphia, March-April, 1987.

Poterba, James. "House Price Dynamics: The Role of Tax Policy and Demography," *Brookings Papers on Economic Activity*, 1991, 2, 143-203.

Sinai, Todd. "Are Tax Reforms Capitalized into House Prices?" in *The Effect of Tax Reform on the Owner-Occupied Housing Market*, MIT Ph.D thesis, May 1997.

Voith, Richard. "Does the U.S. Tax Treatment of Housing Create an Incentive for Exclusionary Zoning?", Federal Reserve Bank of Philadelphia Working Paper, August 1998.